Toronto Metropolitan University

AER615 Aircraft Performance

An Analytical Study of Piper Cherokee Warrior III's Drag Build-Up

Final Report

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ABSTRACT

The analysis in this report was conducted to familiarize the students with the aerodynamic properties consisting of lift and drag on different components of the aircraft. Piper Cherokee Warrior III was used to be studied. The different components selected to be analysed were wing, fuselage, empennage and landing gear. The drag of these components was calculated at different velocities. It was noted that as the lift coefficient increases, the drag coefficient also increases. The equation relating these two terms is parabolic and an equation was derived in the report to represent the curve. Oswald's efficiency was calculated to be 0.85. Drag at different altitudes was also compared and it was observed that the largest amount of drag is produced at 15,000 ft, followed by 10,000 ft and sea-level. The glide performance at an altitude of 10,000 ft was analysed. As velocity increases, the glide ratio reaches a maximum of 16 at an air-speed of 100 kts. As part of the glide performance, the sink rate was also calculated. The sink rate was observed to be at a minimum of 6.5 fps at an airspeed of approximately 55 kts. The analysis in this report was conducted assuming the weight of the total aircraft stays constant during flight and that the flow is turbulent across the fuselage.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

Purpose/Background

The ulterior purpose behind carrying out this project was to introduce the concept of how to analyze and deliver the aerodynamic performance of a specific plane; namely the Piper Cherokee Warrior III, a plane built mainly for the purpose of flight training. It features a semi-monocoque construction making it a minimalist, lightweight, and inexpensive option among the competition of single-engined, piston-powered airplanes, relatively speaking [1].

Aside from this, the method to be used to investigate this aircraft's aerodynamic performance is known as drag-build up, where various plots of the drag of the entire aircraft, as well as each of its components, will be contrasted and compared with lift coefficients and airspeeds, among other variables. Additionally, a plethora of constants and values can be derived from these plots that prove to be imperative to the drag-build up method, such as Oswald's efficiency. In conclusion, the compendium of all this data will provide a comparatively accurate depiction for the aerodynamic performance of the aforementioned aircraft.

Methodology

Supplementary information to make the procedure behind the project smooth and meticulous includes, the Piper Cherokee Warrior III flight manual [2], fluid-dynamic drag insights, aircraft performance formulae, and credible foreign sources. Equations that are simplistic in nature, yet detrimental to understanding the rudimentary knowledge used in this project are wielded to further form more equations that can be explicitly applied to specific cases.

The fundamentals of aerodynamics entails formulae referring to the most unelaborate crux involved with design; that is, factoring for lift, drag, thrust, and weight. As this report deals with the topic of drag-build up, the equations, with reference to these forces, are as follows:

$$C_L = \frac{W}{\frac{1}{2}\rho V_\infty^2 S} \quad (1)$$

Where C_L is the coefficient of lift.

$$C_D = C_{D,0} + kC_L^2 \quad (2)$$

Where C_D is the drag coefficient.

$$C_{D,ind} = \frac{C_L^2}{\pi ARe} \quad (3)$$

Where $C_{D,ind}$ is the induced drag coefficient.

$$D = \frac{1}{2}C_D \rho V_\infty^2 S \quad (4)$$

Where D is the drag.

$$q_\infty = \frac{1}{2}\rho V^2 \tag{5}$$

Where q_∞ is the dynamic pressure.

$$Sink\ Rate = \sqrt{\frac{2WC_D^2}{\rho S C_L^3}} \tag{6}$$

$$Glide\ Ratio = \frac{C_L}{C_D} \tag{7}$$

Aircraft Data

Table 1: Piper Warrior III Data.

Maximum Gross Weight	2325 lbs
Normal Operating Range	45 to 130 KIAS
Wing Dimensions	
Wing Span	35 Ft.
Wing Area	170 Sq.Ft.
Aspect Ratio	7.206
Root Chord	5.25 Ft.
Tip Chord	3.5167 Ft.
Taper Ratio	0.67
Mean Aerodynamic Chord (MAC)	4.44 Ft.
Span Efficiency Factor	0.75
Fuselage Dimensions	
Fuselage Length	23.8 Ft.
Fuselage Area	204.562 Sq.Ft.
Horizontal Stabilizer Dimensions	
Chord	2.5 Ft.
Span	12.979167 Ft.

Assumptions

The following assumptions were made for the ease and simplicity of the calculations. The airspeed ranges from 45 - 130 Knots assuming pressure altitude at sea level and constant 75% power, as it is a prerequisite for normal flight airspeed range at sea level given in the aircraft manual. The critical Re chord was found to be very small as compared to the fuselage length, therefore, a turbulent flow model was assumed along the length of the fuselage. For landing gear drag interference calculations, we assumed the landing gear down for all velocities. Changes in lift and drag with angle of attack were not considered during the calculation as it was not in the scope of this project, which was a cause for error in this project. Horizontal and Vertical stabilizers were assumed to produce no lift and therefore did not contribute towards induced drag. We also ignored the trim drag for empennage as the calculations are out of the scope of this course. The span efficiency factor was considered to be equal to that of a rectangular wing(0.75), even though the wing is tapered. This was a fair assumption because the mean aerodynamic chord was used for all calculations instead of section chords. The effect of dihedral angle on drag was ignored as the concept is not in the scope of this course.

RESULTS

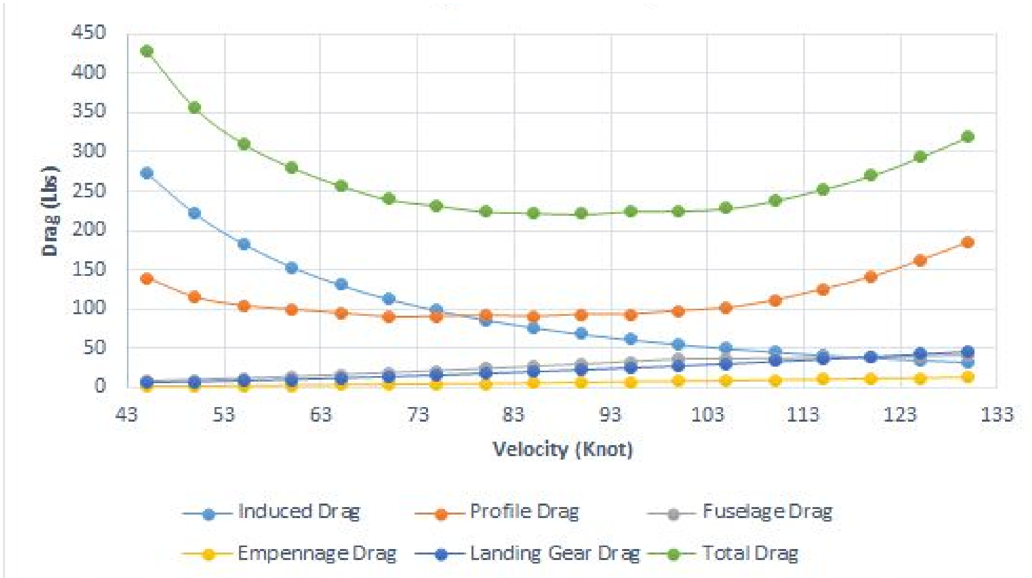

Figure 1: Drag Vs. Velocity for Entire Aircraft and Each Component.

The figure above depicts the relationship between aircraft's true airspeed and drag over the specified variety of components of the aircraft, As seen, the total drag is the sum of Induced and profile drag contributed by wings, empennage drag contributed by horizontal stabilizer, fuselage drag and landing gear drag. The primary source of drag comes from induced and profile drags. Induced drag is seen to be decreasing as the aircraft's speed increases. Whereas, profile drag remains constant till 100 Knots and increases thereafter. Fuselage drag, empennage drag and landing drag remain constant and significantly low.

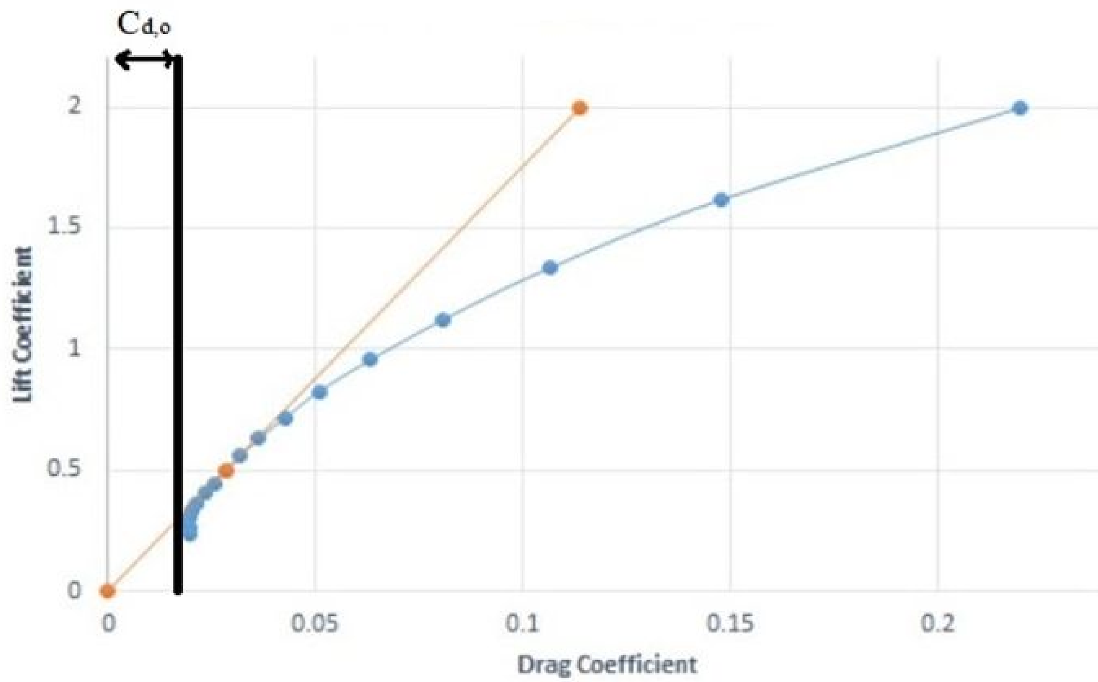

Figure 2: Lift Vs. Drag Coefficient for Entire Aircraft.

The above figure illustrates the relationship between lift and drag coefficients. The tangent to the graph(orange line) represents the point of maximum Lift-to-Drag ratio. The black line represents the minimum C_D value (approximately 0.016).

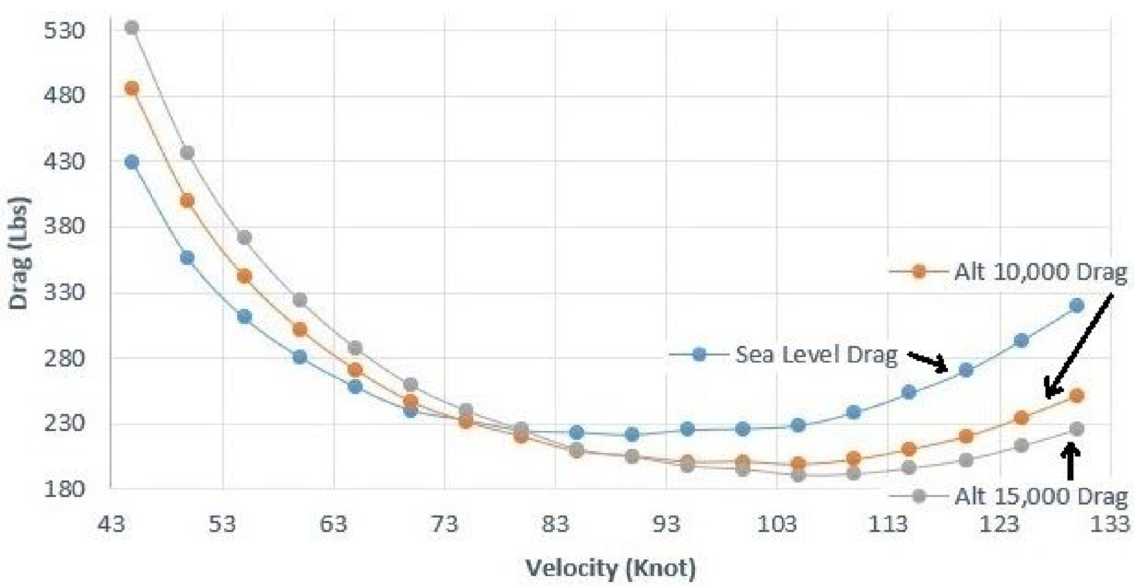

Figure 3: Drag vs. Velocity at Sea Level and Altitudes 10,000 ft and 15,000 ft.

The above figure shows the comparison between total drag buildup at sea level, 10,000 ft and 15,000 ft. As clearly evident from the figure, drag value increases as the altitude increases for the given velocity. This occurs because of decrease in density at higher altitudes.

Effect of Weight on Drag:

As weight increases, the coefficient of lift also increases. This proportionality can be observed from the following equation:

$$C_L = \frac{W}{q*S}$$

As the coefficient of lift increases, the coefficient of drag also increases, which in turn increases drag. This behaviour is described in the following graph:

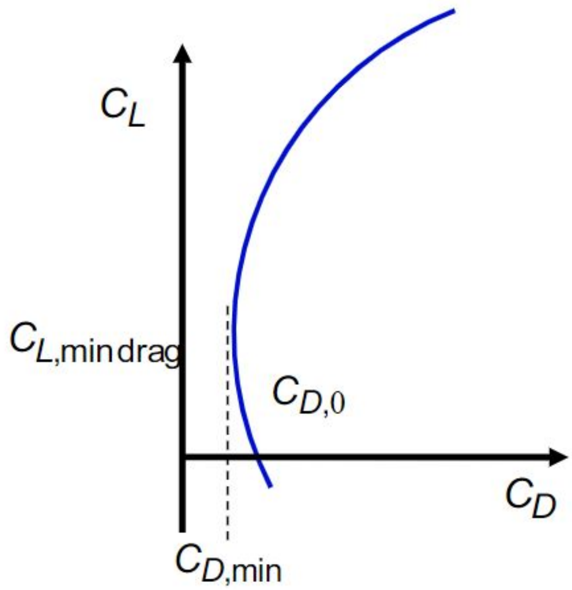

Figure 4: Theoretical C_L Vs. C_D Relationship. [2]

Since Induced drag is proportional to W^2. As the aircraft's weight increases, induced drag increases as well. This observation is made from the following equation of induced drag:

$$D_{ind} = \left(\frac{W}{b}\right)^2 * \frac{1}{q*\pi*e}$$

Glide Performance

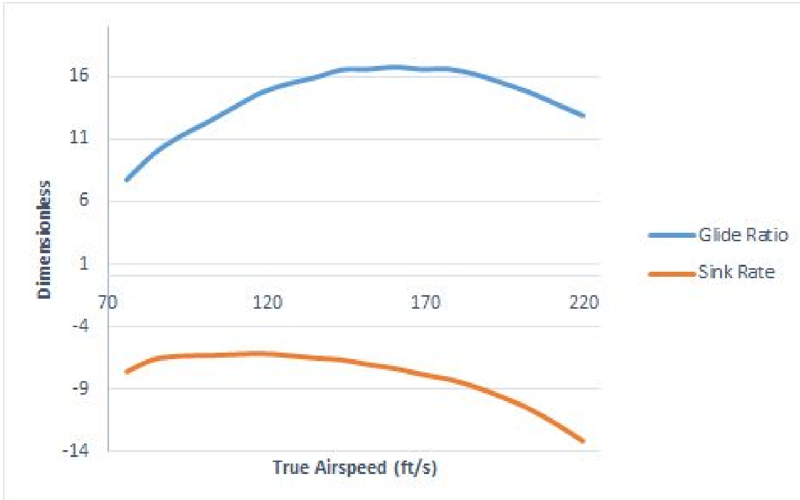

Figure 5: Sink Rate and Glide Ratio vs. True Airspeed

It should be noted that in the figure above, glide ratio and sink rate plots were calculated in two different methods. One method was calculated using the aforementioned parabolic fitting equation to find C_D values. Conversely, the other method used was through using equation (eqn number here) to find C_D values. Both methods yield the same graph, thereby providing a sanity check for the data involved in this case. Sink rate was made negative to accurately represent how a realistic curve would look for the value in question.

Derivation of the Oswald's Efficiency

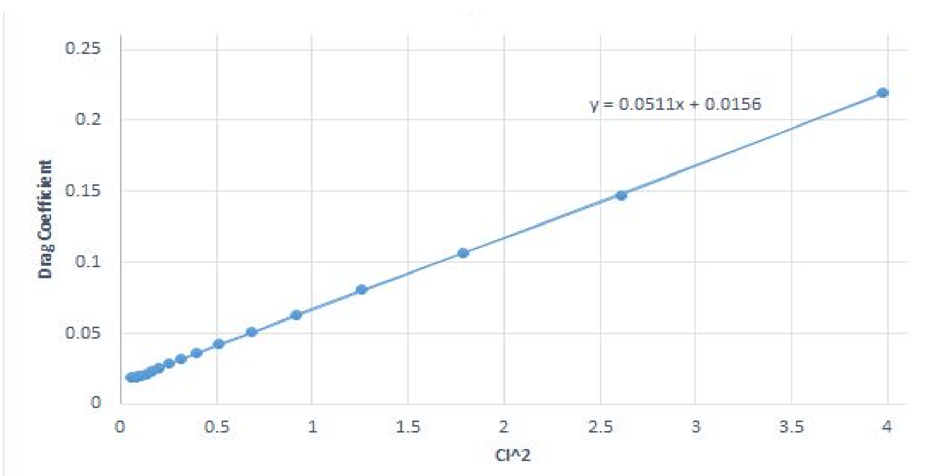

Figure 6: C_L^2 Vs. Drag Coefficient.

Parabolic Fitting Equation: $C_D = C_{D,0} + k\,C_L^2 = 0.0156 + 0.0511\,C_L^2$

$$k = \frac{1}{\pi A R e_o}$$

$$Oswald's\ Efficiency\ (e_o) = \frac{1}{k\pi AR} = \frac{1}{0.0511*\pi*7.206} = 0.85$$

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{\sqrt{\frac{C_{D,o}}{k}}}{2*C_{D,o}} = \frac{\sqrt{\frac{0.0156}{0.0511}}}{2*0.0156} = 17.5$$

From the conceived calculations above, the oswald's efficiency was found to be 0.85. This number is within the range of 0.75 - 0.9

CONCLUSION

Aerodynamic analysis was performed on Piper Cherokee Warrior III. The amount of lift produced by the entire aircraft at various speeds was calculated and analysed. The drag produced by different components of the aircraft was calculated and compared with the amount of lift produced at different air speeds. It was observed that as the drag coefficient increases, the lift coefficient also increases. The relationship between lift coefficient and drag coefficient is parabolic and an equation was derived to represent the relation. Oswald's efficiency factor was calculated to be 0.85. The amount of drag produced by the aircraft at different velocities was compared at different altitudes. It was observed that as velocity increases, total aircraft drag decreases exponentially until the aircraft reaches the speed of 113 Knots. As observed, the maximum drag build-up occurs at the highest altitude, 15,000 ft , followed by 10,000 ft and sea-level conditions. It can be said that as altitude increases, drag increases for the same amount of speed. The glide performance was evaluated for the aircraft at 10,000 ft and 2000 lbs. It was observed that as the velocity increases, glide ratio increases and reaches a maximum of 17.5 at 100 Knots and starts to decline as the speed further increases. To further elaborate on the flight performance, the sink rate of the aircraft was calculated and observed to have a minimum value of 6.5 fps at an air speed of 55 Knots.The drag associated with different components of the aircraft were discussed in the report. The calculations in the report were made assuming that the landing gear is deployed during flight time and that the flow across the fuselage is turbulent at all times. Better and more accurate results are possible by calculating the type of flow and testing the components in wind tunnels. This procedural analysis was in accordance with the expected results and therefore this analysis was conducted properly.

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APPENDIX

Table 2: Aircraft Component Wise Drag Buildup Data.

C_L	$V_\infty\ (ft/s)$	$q_\infty\ (psf)$	$D_{Induced}$	$D_{Profile}$	$D_{Fuselage}$	$D_{Empennage}$	D_{Misc}	D_{Total}
1.99437	75.96	6.8576	283.105	184.868	8.38317	1.921443	5.6106	483.8877
1.61544	84.40	8.4661	231.215	125.139	10.1764	2.326958	6.9266	375.7841
1.33507	92.84	10.244	192.822	114.489	12.1277	2.767447	8.3812	330.5875
1.12183	101.28	12.191	163.622	113.625	14.2349	3.242404	9.9744	304.6985
0.95588	109.72	14.308	140.896	107.293	16.4960	3.751382	11.706	280.1425
0.82420	118.16	16.594	122.865	100.269	18.9091	4.293981	13.576	259.9132
0.71797	126.60	19.049	108.318	100.672	21.4724	4.869841	15.585	252.3563
0.63103	135.04	21.673	96.412	102.111	24.1844	5.478637	17.732	244.4736
0.55897	143.48	24.467	86.545	100.666	27.0435	6.120074	20.018	242.9893
0.49859	151.92	27.430	78.276	103.263	30.0484	6.793883	22.442	241.0826
0.44749	160.36	30.563	71.278	103.522	33.1977	7.499818	25.005	244.8493
0.40386	168.80	33.865	65.304	107.868	36.4901	8.237652	27.707	245.6063
0.36631	177.24	37.336	60.162	111.553	36.9244	9.007180	30.546	248.1929
0.33377	185.68	40.976	55.706	121.455	37.4996	9.808208	33.525	257.9930
0.30538	194.12	44.786	51.818	135.624	38.2143	10.64056	36.642	272.9384
0.28046	202.56	48.765	48.405	150.930	39.0677	11.50408	39.897	289.8049
0.25847	211	52.913	45.394	171.914	40.0586	12.39861	43.291	313.0575
0.23897	219.44	57.231	42.724	194.856	41.1860	13.32401	46.824	338.9141