Toronto Metropolitan University

AER615 Aircraft Performance

An Analytical Study of Piper Cherokee Warrior III’s Rate of Climb

Final Report

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ABSTRACT

This report was used to strengthen prior knowledge of theories and concepts behind aircraft performance, mainly concerning the rate of climb, of a Piper Cherokee. Furthermore, the report also granted it possible to contrast and determine the correlation between experimental values and derived graphs, such as rates of climb at four different altitudes. Aside from this, with respect to the primary goal of this report, several plots were depicted. Major findings, stemming from these plots were that with an increase in altitude, the maximum rate of climb decreased. Additionally, it was also found that at a specific airspeed of 130 knots, the rate of climb of the aircraft remained uniform with changes of altitude.

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INTRODUCTION

Purpose/Background

The purpose of this assignment is to develop a program to estimate the rate of climb of Piper Cherokee at different altitudes. This activity gives an adequate understanding of flight performance parameters such as time takes for climb at different altitudes.

Methodology

The rate of climb and the aircraft performance at 3 different altitudes were analysed.
The parabolic drag coefficient equation was determined using:

$$C_D = C_{D_o} + kC^2{}_L \tag{1}$$

C_{D_o} is then used to calculate the power required. The power required at each of the altitudes was found using the following equation:\

$$P_{req} = C_{D_o} * 0.5 * \rho * V_\infty{}^3 * S + (\frac{W}{b})^2 \frac{2}{\rho * V_\infty \pi * e_o} \tag{2}$$

After the required power is determined, the next step is to calculate Power Available so that the Excess Power can be calculated.

Power Available is given by the following equation:

$$P_{av} = BHP \; * \; \frac{\rho_{alt}}{\rho_{SSL}} * \eta_P \tag{3}$$

Now, the excess power is given by:

$$P_{ex} = P_{av} - P_{req} \tag{4}$$

Subsequently, the rate of climb is given by $RoC = \frac{P_{ex}}{V_\infty}$ and the climb angle is $sin\,\gamma = \frac{RoC}{V_\infty}$.
The results taken from these equations were arranged and the required graphs were plotted in MATLAB.

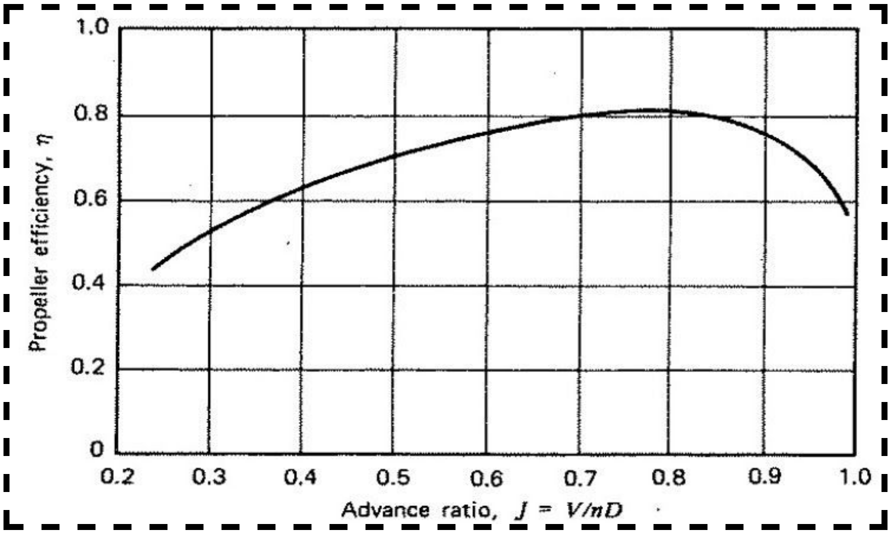

Figure 1: Propeller Efficiency for the Piper Cherokee PA-28.

The above graph was used to find propeller efficiencies at different heights. The propeller efficiency was then used to find power available from equation 3 above.

Aircraft Data

Table 1: Piper Cherokee Data.

Maximum Gross Weight (W)	2400 lbs
Normal Operating Range	30 to 130 KIAS
Parasitic Drag ($C_{d,0}$)	0.0317
$C_{l,max}$	1.45
Wing Area (s)	170 Sq.ft.
Aspect Ratio (AR)	5.71
Oswald's Efficiency Factor (e_o)	0.6
k	0.0929
Propeller Diameter (D)	6.0833 ft.
Engine rps (n)	45 rps
BHP	180 hp

Assumptions

The following assumptions were made before performing the necessary calculations for this project. First off, it was assumed that the aircraft propeller efficiency changes with the increase in altitude. This is because the advanced ratio ‘J’ (related to propeller efficiency through figure 1) is proportional to the velocity of the aircraft. And this velocity was calculated to be different for different altitudes resulting in different propeller efficiencies. The aircraft operating speed range was assumed from 30 KIAS to 130 KIAS as per the operating manual from project 1. The given speed range (in Indicated Airspeed, IAS) was then corrected for density effects giving us the True Air Speed (TAS), which was used for all calculations. The power available from the engine was assumed to be a constant line on Power Vs. Airspeed graph. This is because full power is always, in theory, available no matter the velocity of the aircraft. Lastly, it was a fair assumption to neglect wind effects for climb angle calculations and graphs as it isn't in the scope of this project.

RESULTS

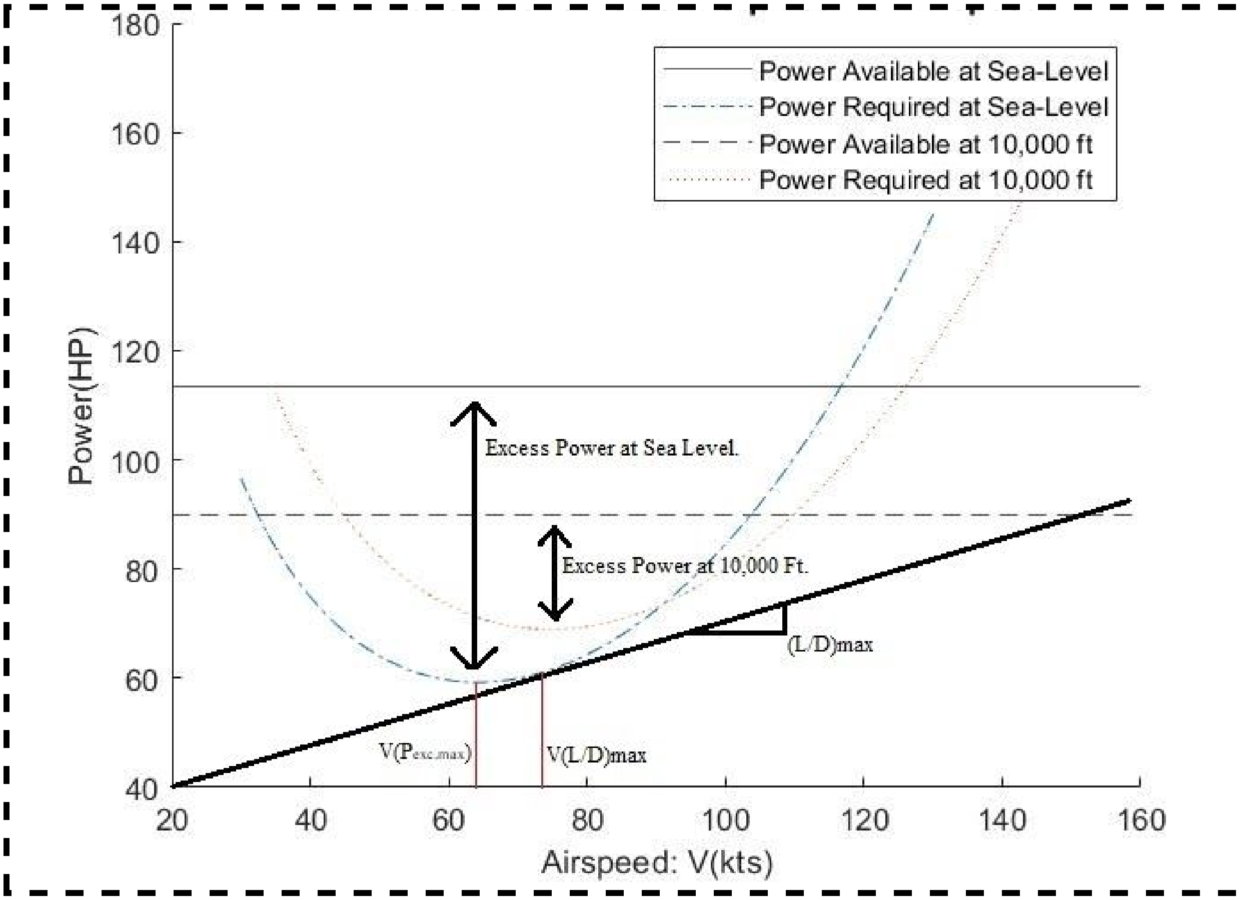

Figure 2: Power Available and Power Required At Sea Level and 10,000 ft.

The above graph illustrates the comparison between Power Available and Power Required curve of Piper Cherokee at sea level and at the altitude of 10,000 ft. As predicted, the Excess Power, difference between power available and power required, decreases with the increase in altitude. This occurs because of reduced levels of density at higher altitudes. As altitude increases, and density decreases, there are less oxygen molecules for the engine intake. Resulting in less power generation by the engine.

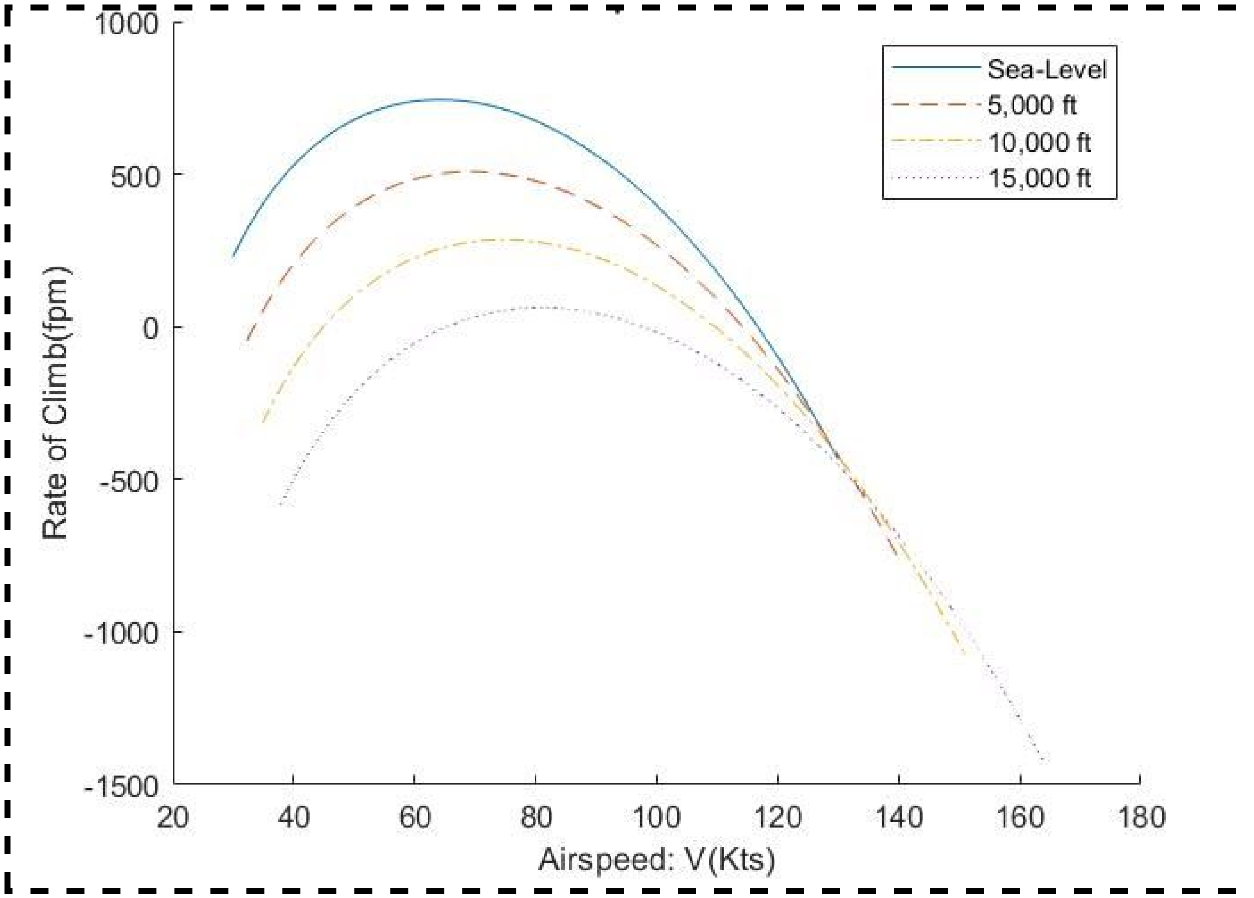

Figure 3: Rate of Climb Vs. True Airspeed at Sea Level, 5,000 ft, 10,000 ft and 15,000 ft.

It can be observed from figure 3 that as altitude increases, the maximum Rate of Climb decreases. Another observation is that while the maximum Rate of Climb decreases with higher altitude, the maximum rate of climb begins to occur at a higher velocity. At an airspeed of 130 kts, the Rate of Climb for all four altitudes is equal. At airspeeds less than 130 kts, the Rate of Climb decreases with increasing altitude. Values for maximum Rate of Climb are presented in table 3 under the appendix section.

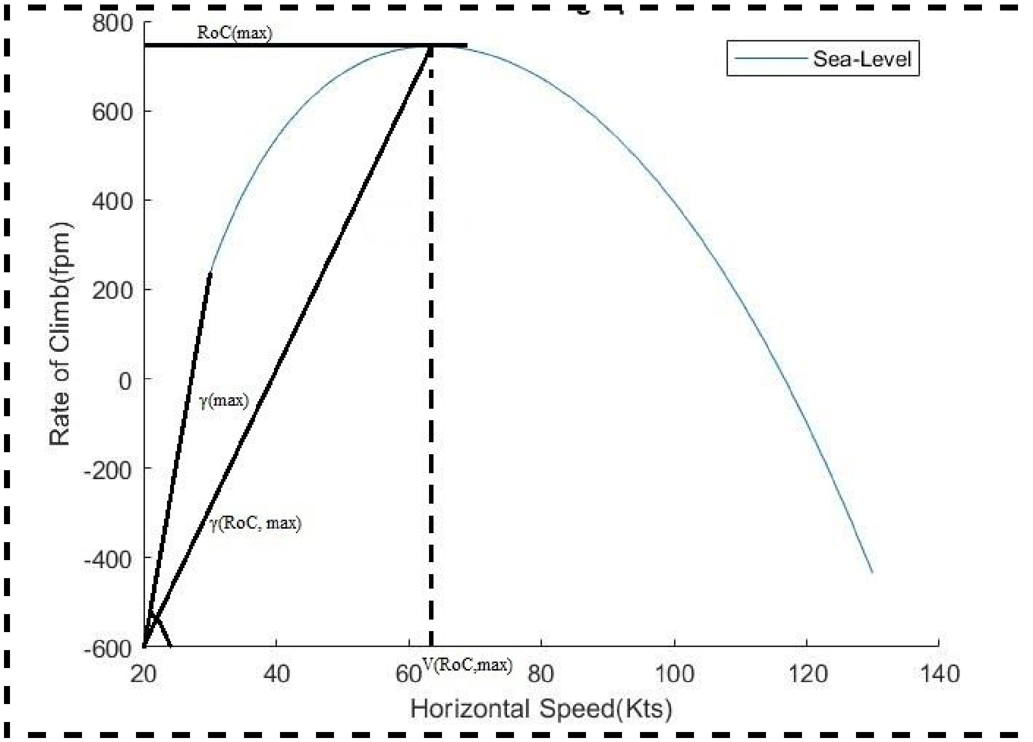

Figure 4: Hodograph of Rate of Climb Vs. Horizontal Speed at Sea Level.

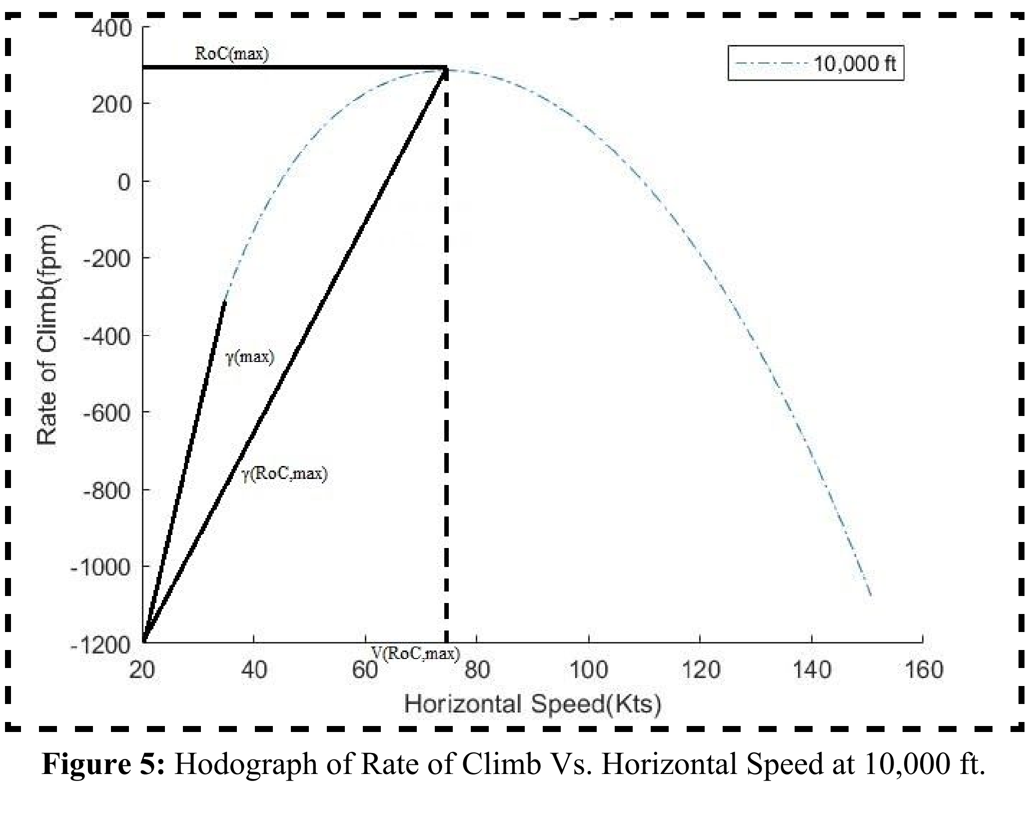

Figure 5: Hodograph of Rate of Climb Vs. Horizontal Speed at 10,000 ft.

In the above two figures, featuring the hodographs of rate of climb at sea level and 10,000 ft, the line joining the origin to a point on the curve, has the length proportional to the flight velocity and the angle this line makes to the horizontal axis (horizontal speed - axis) is the angle of climb. As seen in both figures, the line from the origin and tangent to the curve gives the value of steepest climb angle. These were found to be 7.77° at sea level and 2.28° at 10,000 ft. Hodographs of rate of climb at 5,000 ft and 15,000 ft are presented in the appendix section.

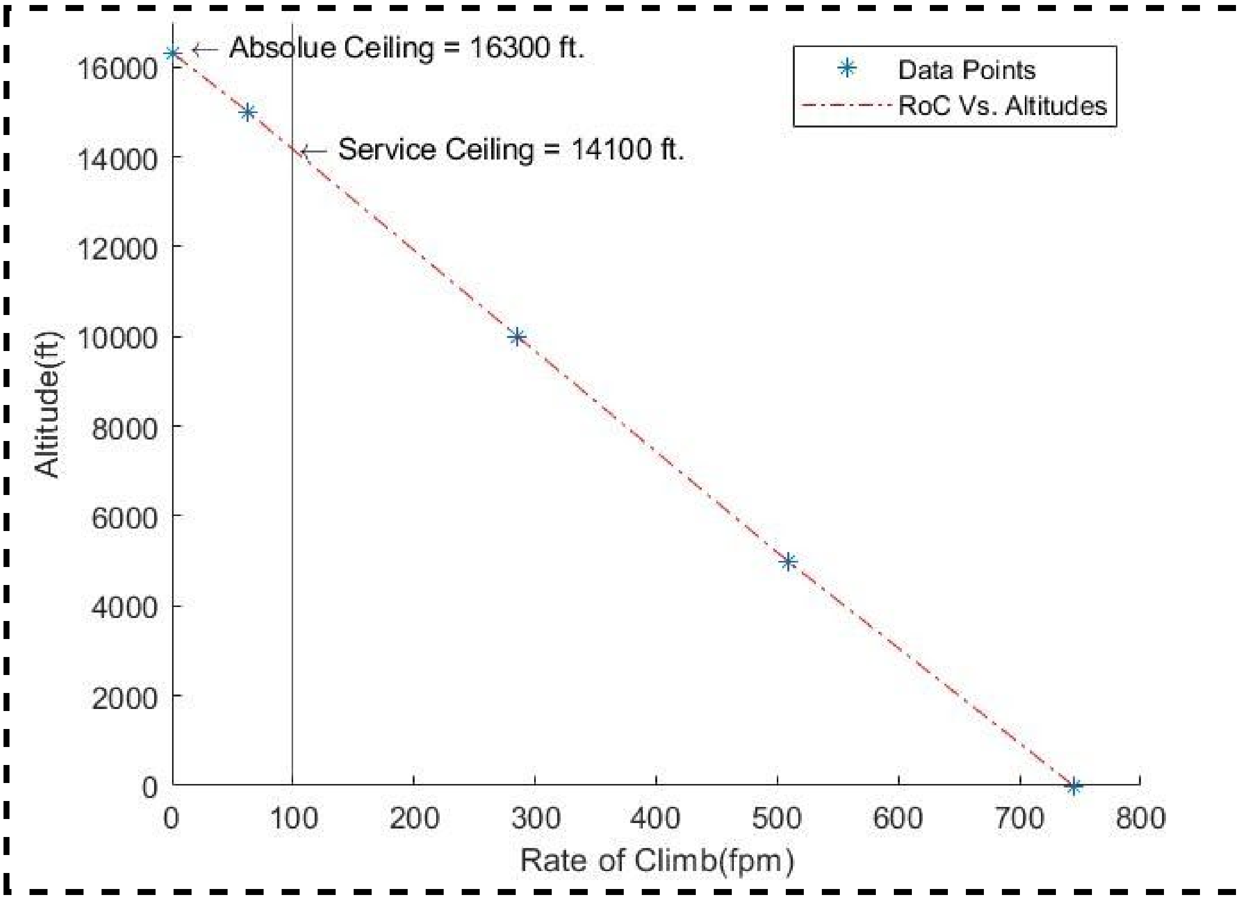

Figure 6: Altitude Vs. Rate of Climb, Absolute and Service Ceiling.

The above graph represents the Absolute Ceiling and Service Ceiling of Piper Cherokee. The aircraft reaches its absolute ceiling when maximum Rate of Climb approaches zero and its service ceiling at a Rate of Climb of 100 ft/min. From the above graph, it is observed that the absolute ceiling is approximately 16,000 ft and the service ceiling is approximately 14,000 ft. The service ceiling obtained from the maximum rate of climb consideration and that obtained from max velocity consideration were very close to each other. Further, the service ceiling of 14,000 ft is close to the actual value of 14,300 ft. [1]

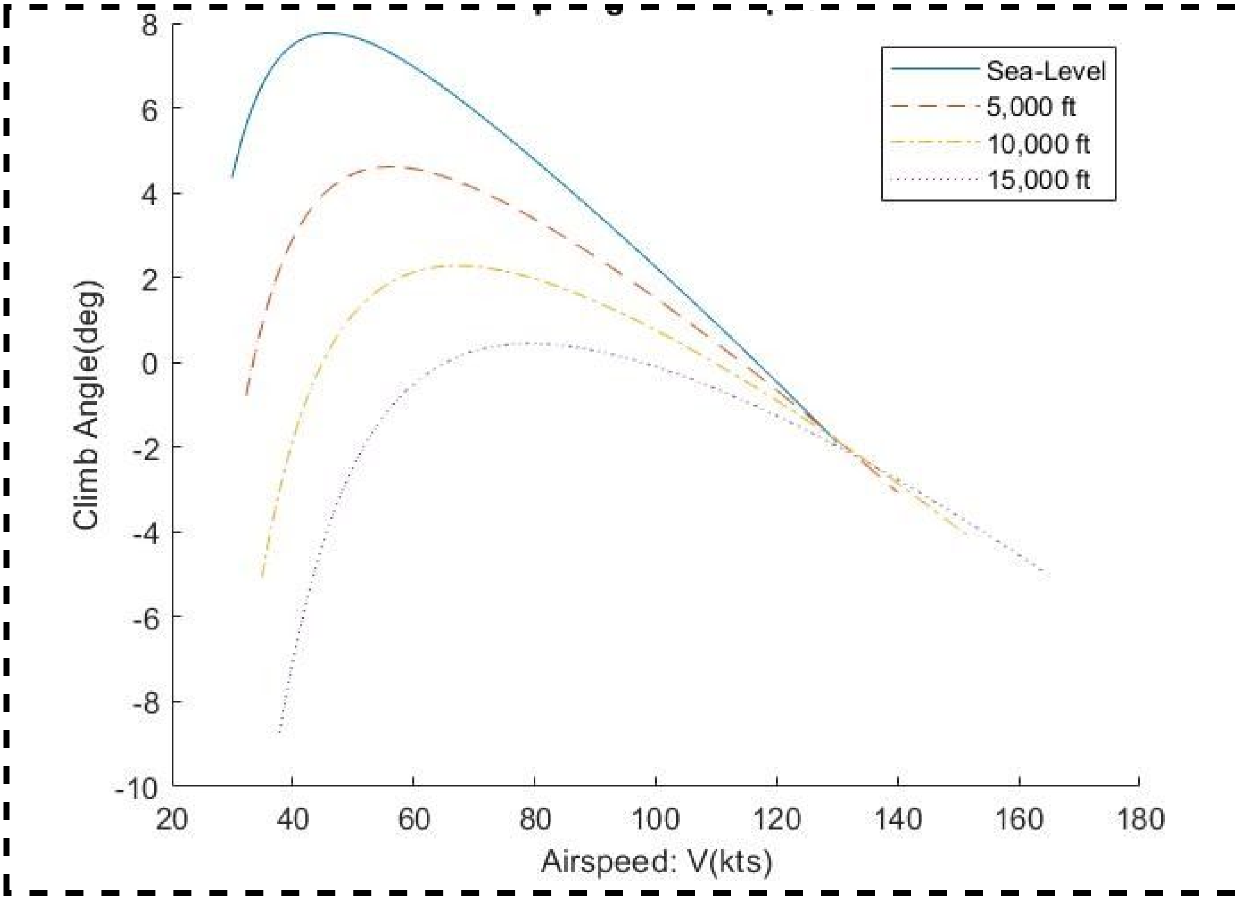

Figure 7: Climb Angle Vs. True Airspeed at Sea Level, 5,000 ft, 10,000 ft and 15,000 ft.

The plots in figure 7 show that as altitude increases, the climb angle increases. Similar to figure 2, at an airspeed of 130 kts, the climb angle and the Rate of Climb are equal at all altitudes. Another difference is that as the altitude increases, the maximum climb angle begins to occur at a higher speed. Aside from this, the plotted steepest climb rate was found to be relatively similar to the climb angle figure above, hence the replacement made. Values for Steepest Climb Angle are presented in table 3 under the appendix section.

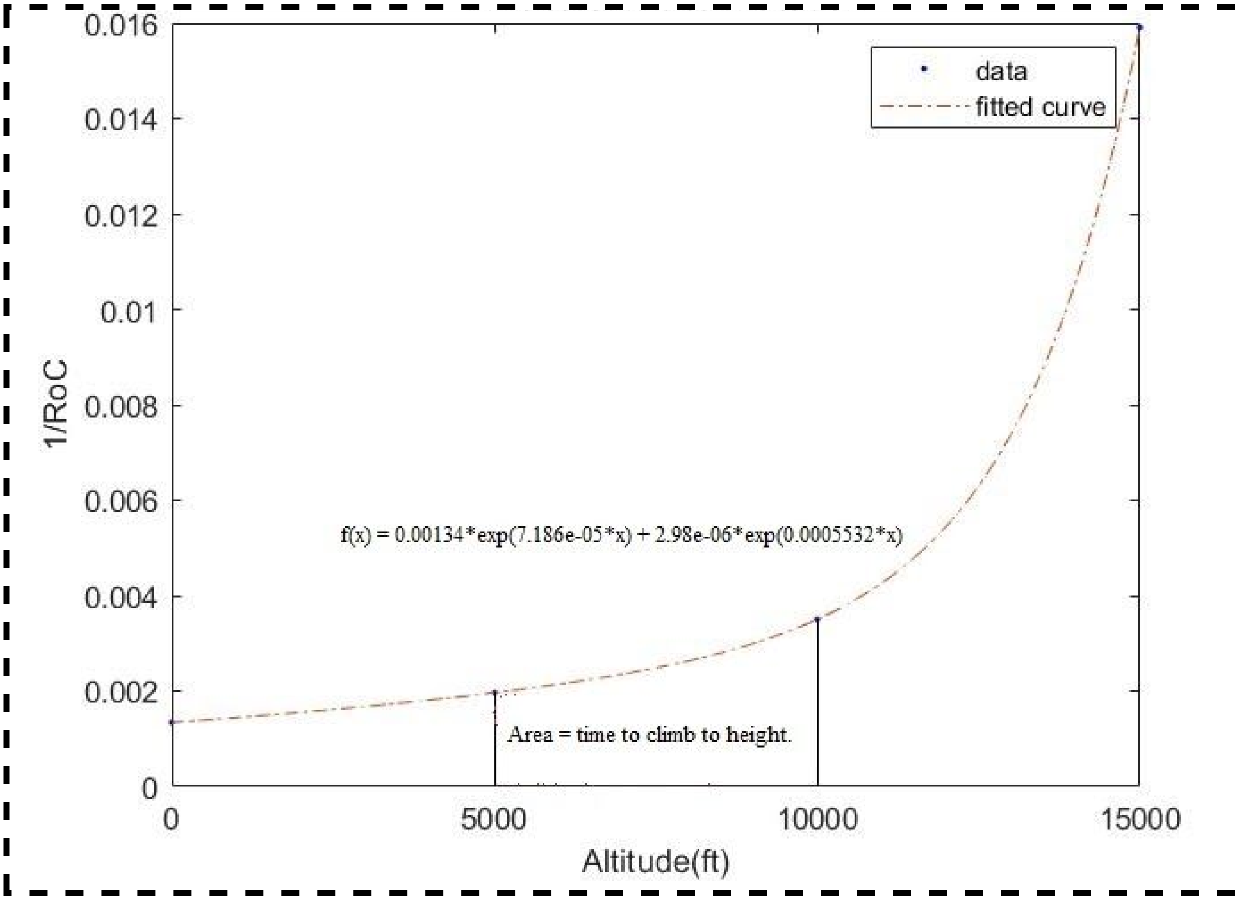

Figure 8: Time to Climb to Altitude Approximation.

The plot in figure 7 shows that at lower altitudes, the Rate of Climb is higher. This means that at lower altitudes, it takes less time to gain altitude than at higher altitudes. This is shown graphically with lower altitudes having smaller areas under the curve. The area under the curve represents the time taken to gain altitude. From the above graph, the lowest altitude range has the smallest time required for a gain in altitude and the highest range of altitude has the largest time required to gain altitude, with the ranges being 0 ft to 5000 ft and 10000 ft to 15000 ft. Calculated Time to climb to altitude is given in the below table.

Table 2: Comparison of Time to Climb from Pilot Handbook Vs. Theoretical.

Altitude (ft)	Actual Time to Climb	Theoretical Time to Climb
5,000	8 min	8.142 min
10,000	25 min	20.964 min
15,000	-	57.775 min

The time to climb values presented above are calculated from sea level to the given altitudes.

CONCLUSION

In summary, through provided data and observations made, a program was made to estimate the rate of climb of a Piper Cherokee at different altitudes for standard conditions. This method of estimation was produced using MATLAB, where figures were plotted over relevant scales, all relating to climb rate. Additional regions of interest were annexed to plots, such as figure 3, for the purpose of elaboration and detailing. Apart from this, the rate of climb was graphed against airspeed, horizontal speed, and altitude, to put simply. Important findings pertaining to the report include, the general relationship between the rate of climb and altitude (observed to be inverse) and a converging point on figure 2 where all plots coincide (130 knots).

Reflecting on the purpose of this report, much was learned about the performance of the Piper Cherokee. Critical analysis was carried out on the rate of climb among other properties that portray a concise representation of this aircraft's behaviour relative to aforementioned variables. Nevertheless, an improvement that could be made to the coding and iterative process are avoiding estimations made when referring to the propellor efficiency graph, such as implementing a way to accurately interpolate the propellor efficiency from the advance ratio calculation. In conclusion, the report proved to be effective in cementing a foundation in aircraft performance, through the thorough focus on an aircraft's rate of climb.

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[4] “Appendix A: Volumes, Surface and Wetted Areas,” Wiley Online Library, 27-May-2013. [Online]. Available: https://onlinelibrary.wiley.com/doi/pdf/10.1002/9781118568101.app1. [Accessed: 03-Marb-2021].

APPENDIX

Table 3: Experimental Aircraft Data.

Altitude (ft)	Advance Ratio, J	Propellor Efficiency, η_p	Excess Power, P_{ex} (hp)	Maximum Rate of Climb (fpm)	Steepest Climb Angle (deg)
Sea Level	0.3958	0.630	29777	12.4071	7.7662
5000	0.4264	0.650	20333	8.4723	4.6128
10000	0.4606	0.675	14190	4.7578	2.2778
15000	0.4990	0.700	25138	1.0474	0.4439

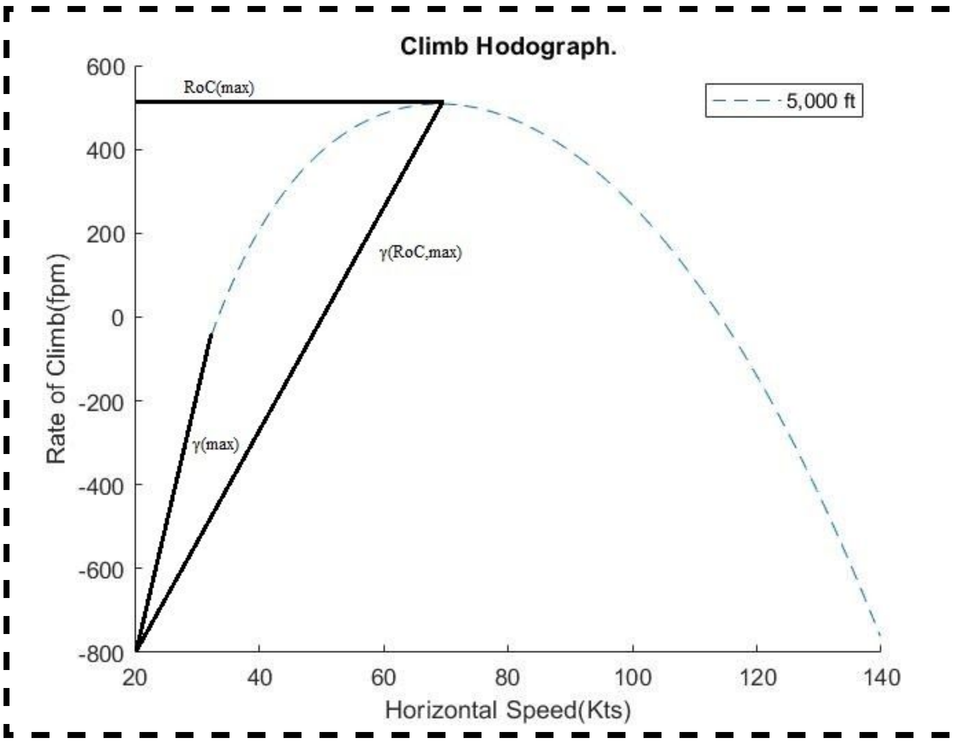

Figure 9: Hodograph of Rate of Climb Vs. Horizontal Speed at 5,000 ft.

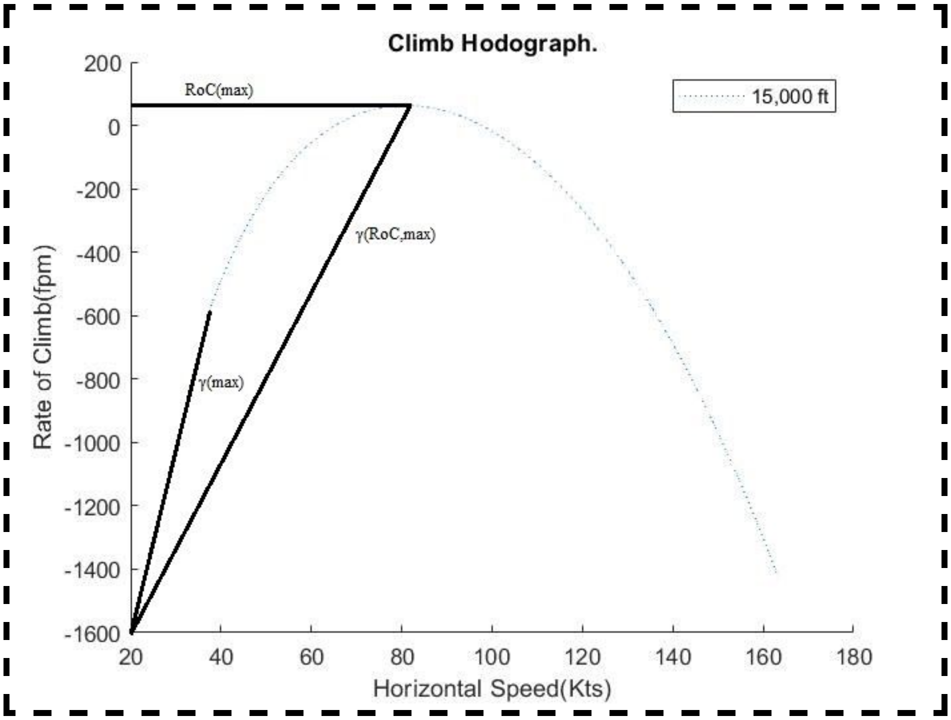

Figure 10: Hodograph of Rate of Climb Vs. Horizontal Speed at 15,000 ft.