Toronto Metropolitan University

# AER615 Aircraft Performance

## An Analytical Study of Boeing 737Er's Take-Off Performance

Final Report

**Authors:** Aman Gilani, Dylan Callaghan, Ibrahim Khan

## ABSTRACT

This report was used to strengthen prior knowledge of theories and concepts behind aircraft performance, mainly concerning takeoff properties, of a Boeing 767ER. Furthermore, the report also granted it possible to contrast and determine the correlation between experimental values and derived graphs, such as ground roll distance and speed, with and without wind inputs. Aside from this, with respect to the primary goal of this report, several plots were depicted. Major findings stemming from these plots was the derived takeoff distance being equated to the actual takeoff distance of a Boeing 767ER, both being approximately 8860 ft [1], thereby proving the report to be relatively substantiated in its premises. The report also finds the ground roll distance without head-wind is approximately 6200ft and with a head-wind of 15 kts, the ground-roll distance is reduced by 300ft to reach an airspeed of 160 kts. A scenario where one engine is inoperable is also analysed in the report. The ground distance was drastically increased till a velocity of 160 kts was reached. This is because enough thrust is not generated and it takes a longer time to reach an airspeed of 160 kts. The reasons are provided in depth in the report.

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# INTRODUCTION

## Purpose

The purpose of this project is to develop a program to determine the ground-roll distance, velocity and acceleration as a function of time. The aircraft analysed for its take-off process in this report is Boeing 767ER.

## Background

The takeoff process consists of 3 main stages namely ground roll, air distance and climb-out. In the figure below, the airplane accelerates till it reaches a rotation velocity. The aircraft rotates till it lifts-off with a lift-off velocity. The take-off distance consists of the take-off and air-distance before the obstacle is cleared. As it can be observed, if enough thrust is not generated and the lift-off speed is not reached as predicted, the aircraft would not clear the obstacle successfully.

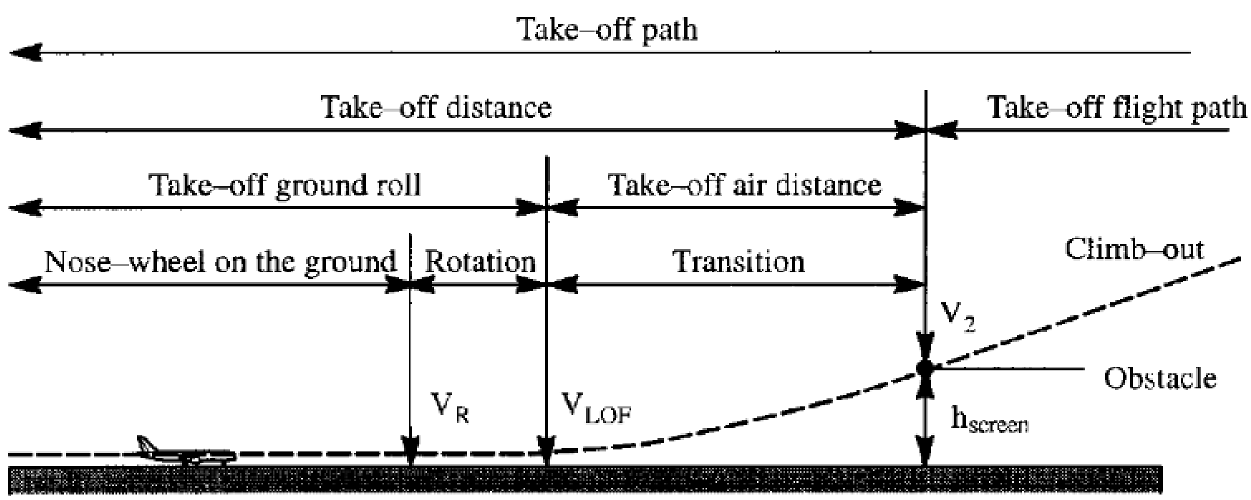


**Figure 1:** Geometry of Take-off process. [2]

To conduct the necessary aircraft performance on Boeing 767ER, the equation for drag coefficient with the gear and flaps down is provided and it is as follows:

$$C_d = 0.0413 + 0.0576 * c_l^2 \quad (1)$$

Using the above drag coefficient equation, the drag at different stages of lift-off can be calculated analytically by calculating the respective lift coefficients. The lift coefficient can be calculated as follows:

$$C_l = \frac{2W}{\rho_{sl} * V^2 * S} \quad (2)$$

The velocity at different stages needs to be determined before attempting to solve for the lift coefficient. The velocity at lift-off is given by:

$$V_{lof} = 1.1 * V_s \quad (3)$$

The velocity at stage 2 is given by:

$$V_2 = 1.2 * V_s \quad (4)$$

Where Vs is calculated using $C_{l_{max}}$ and is as follows:

$$V_s = \sqrt{\frac{2*w}{\rho*s*C_{l,max}}} \quad (5)$$

After the drag is determined, the thrust at different stages, using the velocity at different stages, can now be evaluated using the equation provided below.

$$T = 1000[55.6 - 4.60\left(\frac{V}{100}\right) + 0.357\left(\frac{V}{100}\right)^2]$$

(6)

The resultant forces at different stages can now be processed through the following set of equations:

$F_o = T - \mu_g W$, where $\mu_g$ is the ground friction coefficient. (7)

$$F_{lof} = T_{lof} - D_{lof} \quad (8)$$

$$F_{v2} = T_{v2} - D_{v2} \quad (9)$$

Lastly, the take-off distance is found by summing the ground distance with the airborne distance and is as follows:

$$S_{TO} = S_G + S_A \quad (10)$$

Where $S_{TO} = Take - off\ distance$, $S_G = Ground\ Distance$ and $S_A = Airborne\ distance$.

## Assumptions

1. For this project, we assumed a dry concrete runway with a constant friction coefficient ($\mu_g$).
2. The runway has a zero runway inclination angle(gradient).
3. We also assumed that the change in weight during takeoff and climb is negligible because the fuel used is very small relative to the overall fuel, which directly affects the overall weight.
4. For this project, we used B767 200-ER data for all the calculations. All the calculations were made assuming FAR 25 guidelines.
5. $C_{l,max}$ for B767 200-ER was found to be 2.03, and $V_2$ was found to be 168 Kts.
6. It was also assumed that airspeed and Flight Path(γ) are independent variables. The assumption proves valid because airspeed does not change with change in (γ).

7. The ratio of nose gear reaction to weight $\frac{N_n}{W}$ was found to be 0.08 from the textbook to calculate the ground roll after 3 sec reaction for the balanced field length.
8. Breaking friction coefficient($\mu_{g,break}$)was found to be 0.7 based on the dry concrete runway assumed above.

## Aircraft Data for Analytical Approach

The table below depicts provided and derived criteria to be used in various formulae and conditions involved with the iterative estimation used in this report.

**Table 1:** Aircraft Data.

| | |
|---|---|
| Weight($W$) | 387000 lbs |
| WingArea($S$) | 3084 $ft^2$ |
| Thrust($T$) | 91560 lbs |
| $\mu_g$ | 0.025 |
| $c_{l,g}$ | 1 |
| $c_d$ | $0.0413 + 0.0576 * c_l^2$ |
| $C_{l,max}$ | 2.03 |
| Obstacle Height($h_{obs}$) | 35 ft |

## RESULTS

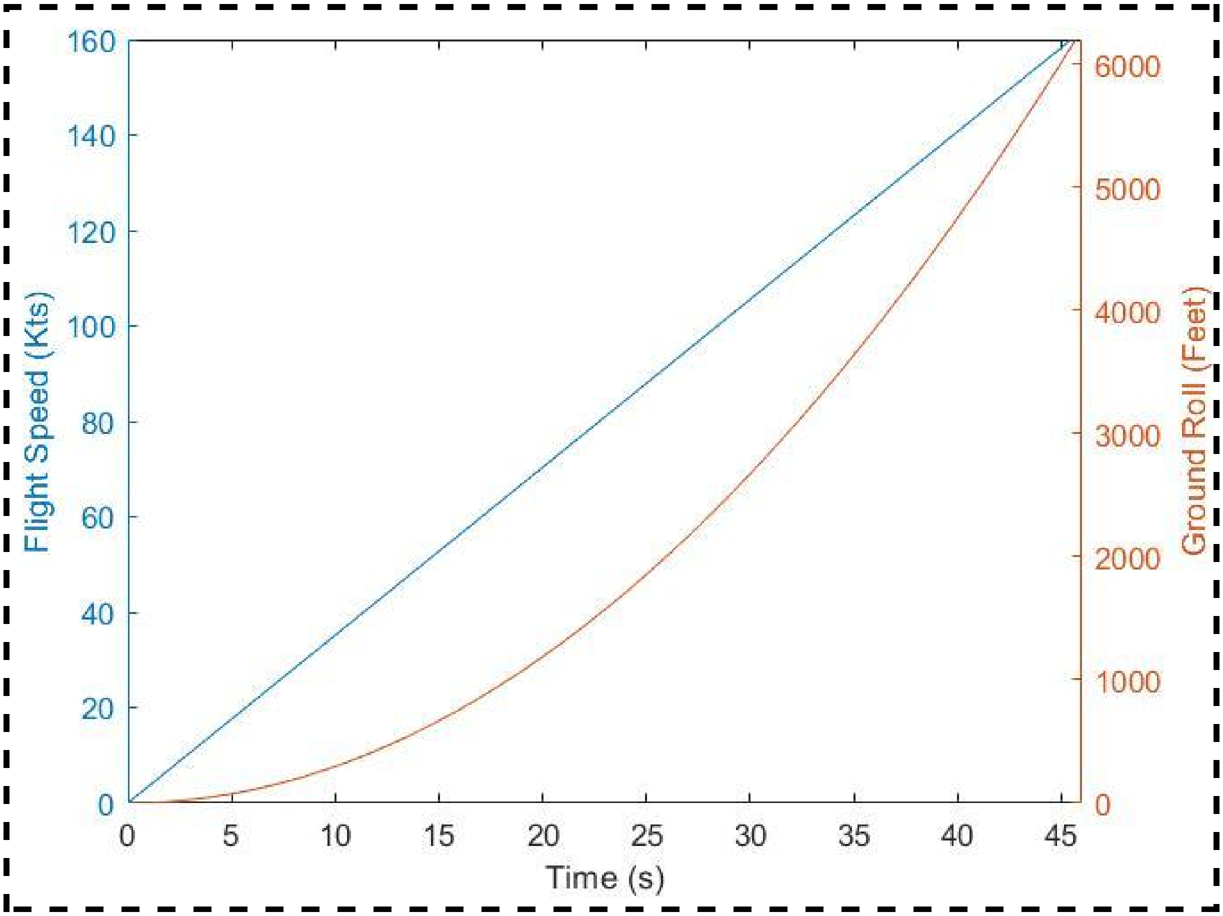


**Figure 2:** Ground Roll Distance and Speed Vs. Time.

The above graph illustrates a linear relationship between ground roll flight speed and time whereas a parabolic relationship between ground roll distance and time. The maximum ground roll distance occurs at liftoff speed of 160 Kts. This distance was found to be 6,200 ft. The results are very similar to the calculated ground roll distance, 6,175 ft, from analytical approximation method.

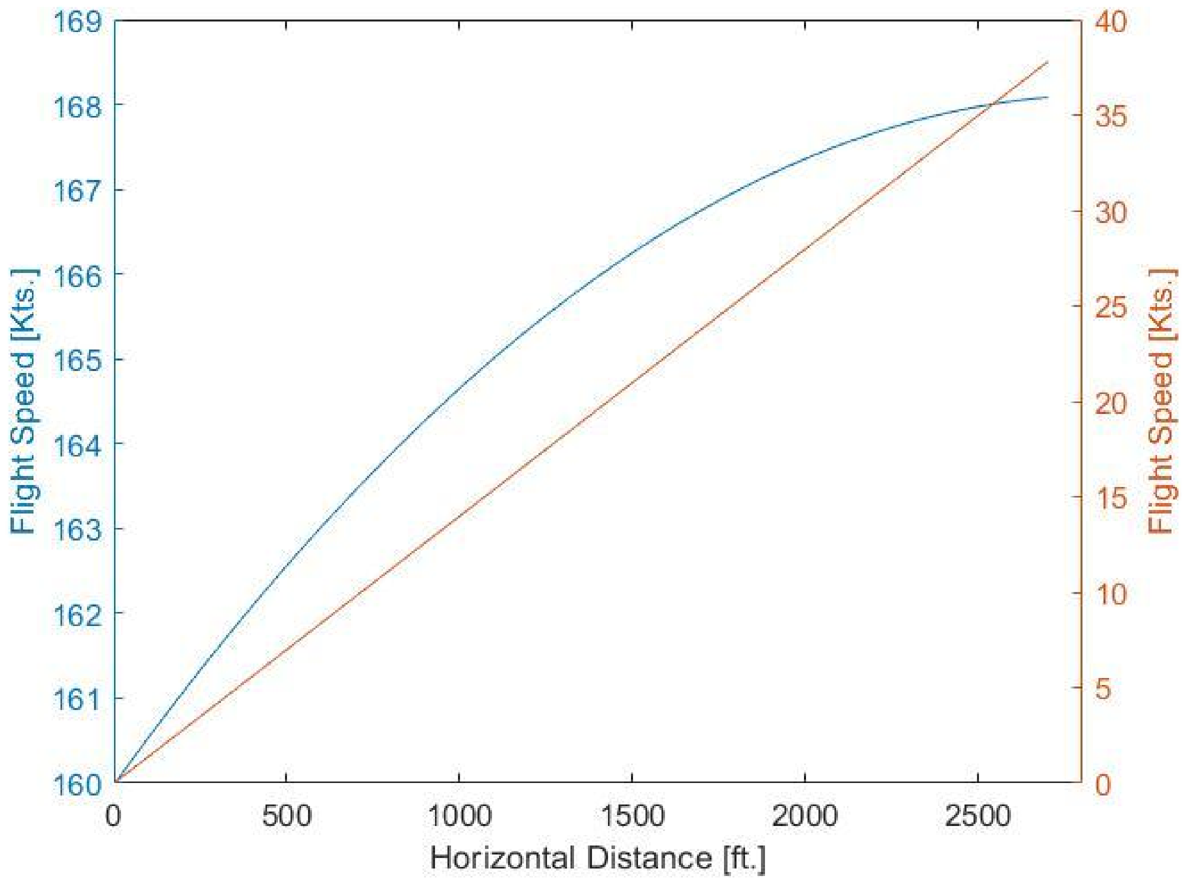


**Figure 3:** Speed and Altitude Vs. Ground Distance During Initial Climb Out Until Clearing.

The above graph represents a linear relationship between altitude and ground distance during initial climbout, and a non-linear decreasing relationship between flight speed and ground distance. The total airborne distance was calculated to be around 2500 ft, which is approximately equal to the airborne distance of 2692.1 ft, calculated from analytical approximation method.

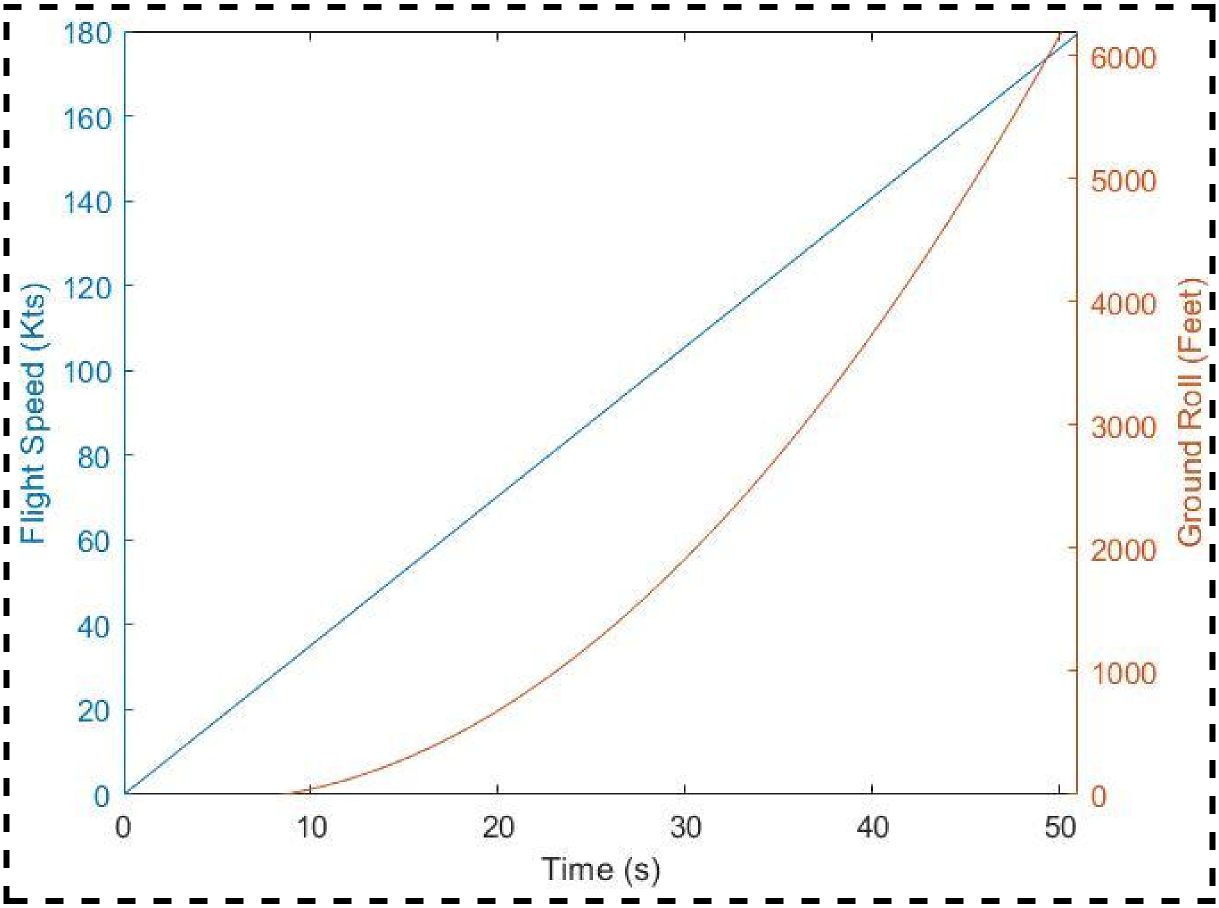


**Figure 4:** Ground Roll Distance and Speed Vs. Time, 15 Kts. Headwind.

The above graphs represent a linear relationship between ground roll flight speed and time whereas parabolic relationship between ground roll distance and time at 15 Kts headwind. This graph is different from the one in figure 1, as it accounts for the 15 Kts headwind on the aircraft. Theoretically, a lower ground speed and a shorter run is needed for the plane to become airborne in case of any headwind. This theory is confirmed from the graph in figure 3. As we can see the ground roll distance is reduced by approximately 300 ft. The ground roll distance is 5,850 ft. There is no decrease in the flight speed as predicted because we assumed the aircraft would lift off at 160 Kts no matter the headwind. This wasn't a fair assumption as it is not theoretically correct but it made our calculations much easier. Theoretically the lift off speed should be reduced by the headwind, in this case the lift off speed was calculated to be 145 Kts.

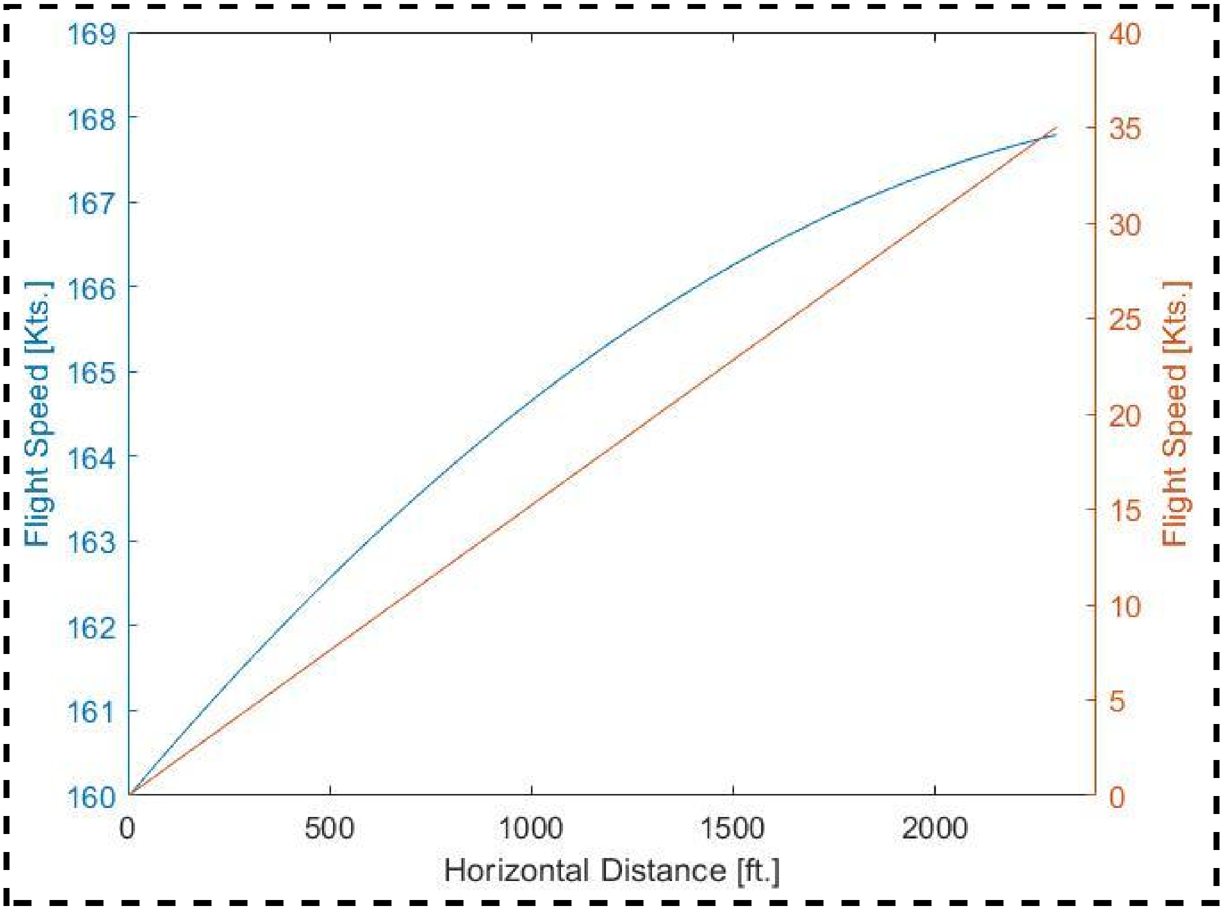


**Figure 5:** Speed and Altitude Vs. Ground Distance During Initial Climb Out Until Clearing, 15 Kts. Headwind.

The above figure shows a linear relationship between altitude and ground distance during initial climbout, and a non-linear decreasing relationship between flight speed and ground distance. The graph behaved predictably as compared to figure 2. The only difference is airborne distance covered by the aircraft. In the case of 15 kts. Headwind, the airborne distance was calculated to be approximately 2261 ft, whereas without any wind the airborne distance was approximately 2500 ft. The difference in airborne distances was expected as the headwind helps the aircraft takeoff and reach the desired velocity much faster. We witnessed the same when calculating and comparing ground roll distances with and without the headwind.

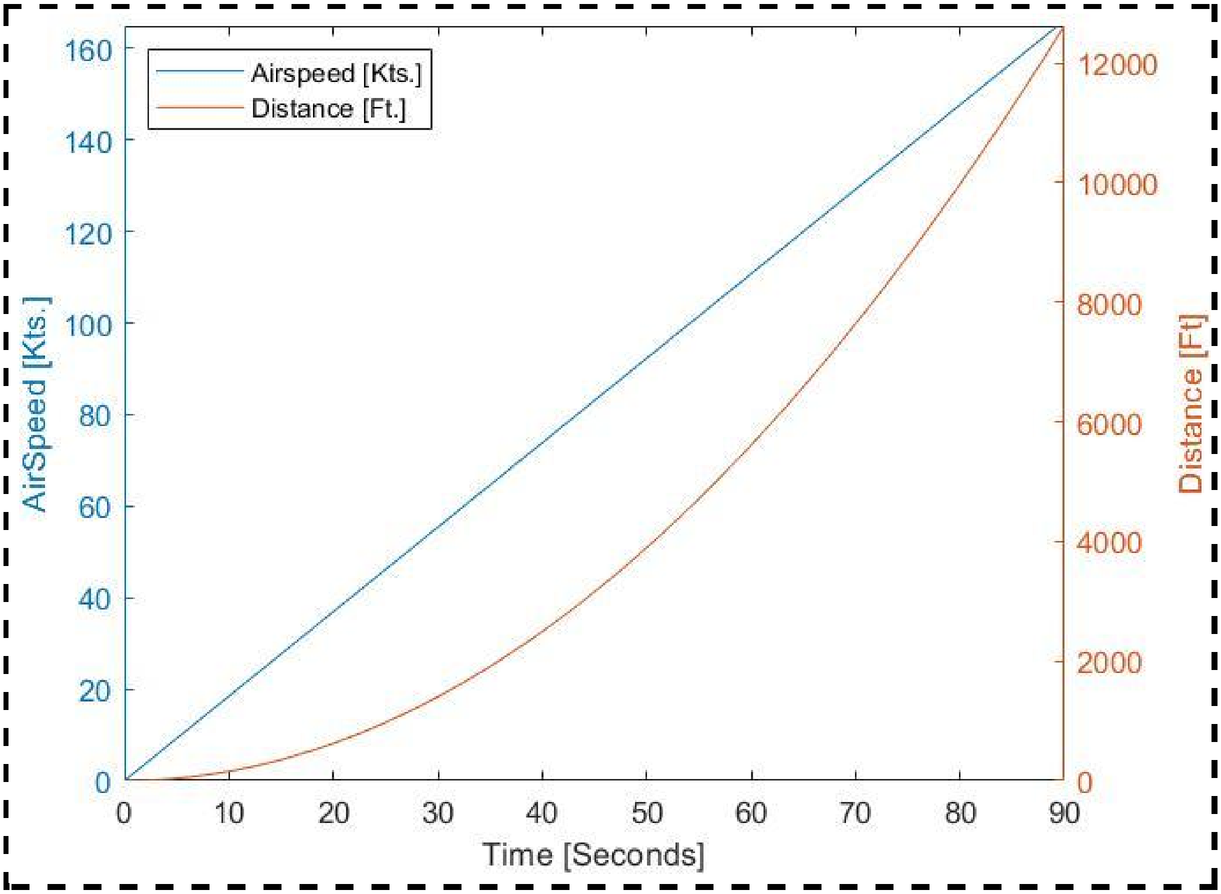


**Figure 6:** Ground Roll Distance and Speed Vs. Time, OEI at 80 Kts.

The above figure shows a linear relationship between airspeed and time which is in consistency with the other graphs. There is a parabolic relationship between the distance and time. Since one of the engines is in-operative, the zero-lift coefficient was assumed to be 10% higher. It can be observed that the distance covered at 160 kts is around 12,100 ft. This makes sense because since in this scenario, one of the engines is inoperable, the aircraft takes a longer time to reach 160 kts and due to this longer time required, the distance covered is also larger than the above scenarios.

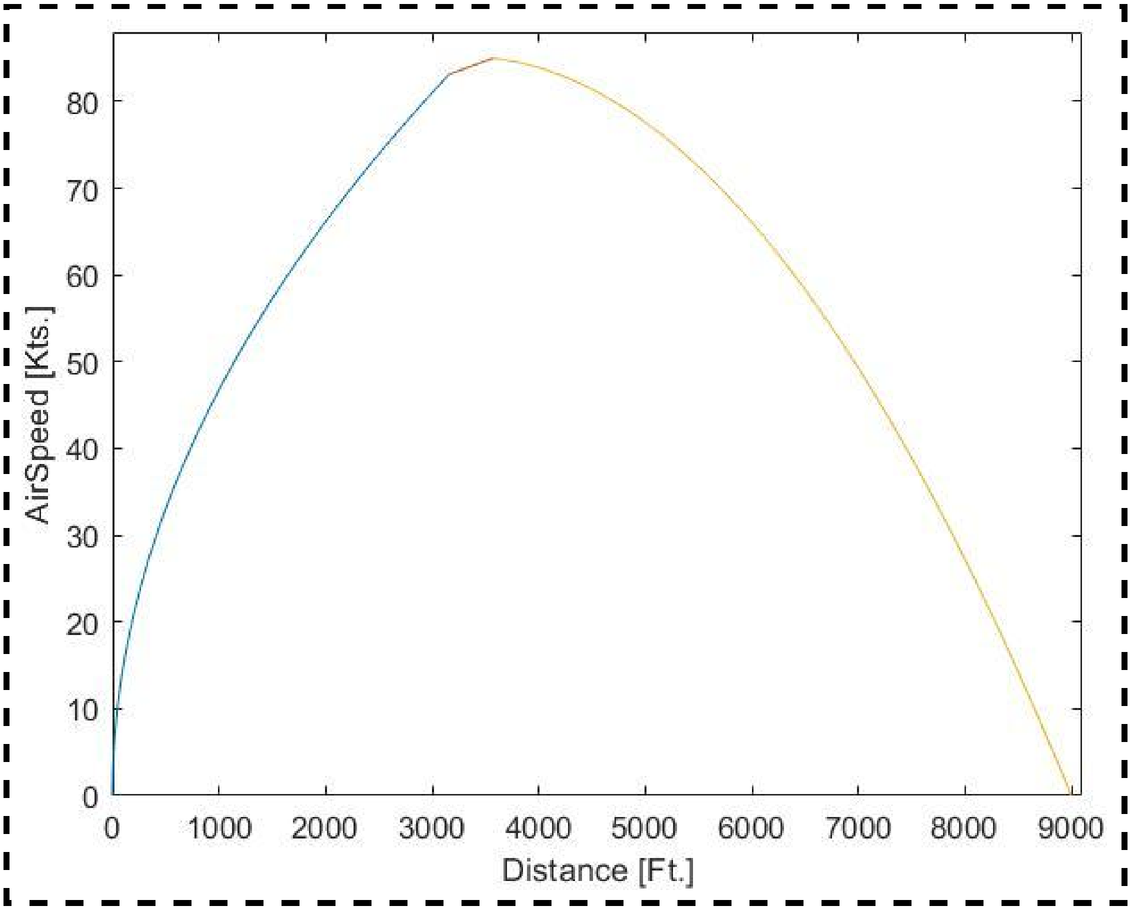


**Figure 7:** Balanced Field Length.

The above graph represents the balanced field length when one of the engines is made inoperable at an airspeed of 80kts. Before the engine is made inoperable at 80 kts, the airspeed increases quadratically with a slower increase at each time interval. As soon as the engine is made inoperable, the airspeed increases linearly and slower than when the engine was operable. After 3 seconds braking is initiated, the airspeed decreases quadratically. It can be observed that the Balanced Field Length from the above graph is 9000 ft. The actual value for balanced field length is 9165 ft and the percentage difference between the two is 1.8 %. The graphed result is very close to the actual value and therefore is a good representation of Balanced Field Length.

## Analytical Approach

**Table 2:** Analytical Approach Results.

| | |
|---|---|
| Stall Speed($V_s$) | $228\frac{ft}{s} = 135\ Kts.$ |
| Lift-off Speed($V_{lof}$) | $250.87\frac{ft}{s} = 148.62\ Kts.$ |
| Speed at Climb Out($V_2$) | $273.7\frac{ft}{s} = 162\ Kts.$ |
| $C_{l,lof}$ | 1.68 |
| $C_{l,v2}$ | 1.41 |
| $D_{lof}$ | 47096.50 lbs |
| $D_{v2}$ | 42751.74 lbs |
| $F_o$ | 81885.00 lbs |
| $F_{lof}$ | 44463.50 lbs |
| $F_{v2}$ | 48808.30 lbs |
| Ground-Roll Distance($S_g$) | 6171.5 ft |
| Flight Path(γ) | 0.01123 rad |
| Airborn Distance($S_a$) | 2692.1 ft |
| Take-off Distance($S_{T/O}$) | 8864 ft. |

Sample calculations are in the appendix section.

## CONCLUSION

In summary, through provided data and observations made, a program was made to estimate the ground-roll distance, velocity, and acceleration of a Boeing 767ER with different conditions, as a function of time. This method of estimation was produced using MATLAB, where figures were plotted over relevant scales, all relating to values commonly associated with takeoff performance. Apart from this, the actual takeoff distance was 8860ft and our analytical approach resulted in a takeoff distance of 8864ft, thus proving the accuracy and efficacy of our method. To strengthen the data, online data also shows the take-off distance is 8858.3. It was also observed that the ground-roll distance in the absence of head-wind was larger than with a head-wind of 15 kts. This is inline with theory since the headwind assists in contributing to the overall lift of the aircraft. The aircraft rotates at a lower amount of ground distance and therefore lifts off faster with a headwind.

Reflecting on the purpose of this report, much was learned about the performance of the Boeing 767ER. Critical analysis was carried out on takeoff performance among other properties that portray a concise representation of this aircraft's behaviour relative to aforementioned variables. Nevertheless, an improvement that could be made to the coding and iterative process is avoiding the assumption of $\gamma$increasing linearly for airborne plots, as in reality it increases non-linearly and stays constant once final $\gamma$ is reached. Additionally, the assumption that all landing gears rose at the same time was made, which is also contrary to reality, as the main landing gears are preceded by the nose gear. In conclusion, the report proved to be effective in cementing a foundation in aircraft performance, through the thorough focus on an aircraft's takeoff process.

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## APPENDIX

$$C_d = 0.0413 + 0.0576 * c_l^2$$

$$V_s = \sqrt{\frac{2*w}{\rho*s*C_{l,max}}} = \sqrt{\frac{2*387,000}{0.002377*3084*2.03}} = 228.1\ \frac{ft}{s}$$

$$V_{lof} = 1.1 * V_s = 1.1 * 228.1 = 250.87\ \frac{ft}{s}$$

$$V_2 = 1.2 * V_s = 1.2 * 228.1 = 273.7\ \frac{ft}{s}$$

$$C_{l,lof} = \frac{2*w}{\rho*s*V_{lof}^2} = \frac{2*387000}{0.002377*3084*250.87^2} = 1.677$$

$$C_{l,V_2} = \frac{2*w}{\rho*s*V_2^2} = \frac{2*387000}{0.002377*3084*273.7^2} = 1.4094$$

$$D_{lof} = (0.0413 + 0.0576 * c_{l,lof}^2) * 0.5 * \rho * V_{lof}^2 * s =$$
$$= (0.0413 + 0.0576 * 1.677) * 0.5 * 0.002377 * 250.87^2 * 3084 = 47096.5\ lbs$$

$$D_{V_2} = (0.0413 + 0.0576 * c_{l,V_2}^2) * 0.5 * \rho * V_2^2 * s =$$
$$= (0.0413 + 0.0576 * 1.4094) * 0.5 * 0.002377 * 273.7^2 * 3084 = 42751.74\ lbs$$

$$F_o = T - \mu_g * w = 91560 - 0.025 * 387000 = 81885\ lbs$$

$$F_{lof} = T_{lof} - D_{lof} = 91560 - 47096.5 = 44463.5\ lbs$$

$$F_{V_2} = T_{V_2} - D_{V_2} = 91560 - 42751.74 = 48808.3\ lbs$$

$$S_g = \frac{w}{2*g} * \frac{V_{lof}^2}{F_o - F_{lof}} * ln(\frac{F_o}{F_{lof}})$$
$$= \frac{387000}{2*32.2} * \frac{250.87^2}{81885-44463.5} * ln(\frac{81885}{44463.5}) = 6171.5\ ft$$

$$\gamma = \frac{1}{w} * (F_{V_2} - F_{lof}) = \frac{1}{387000} * (48808.3 - 44463.5) = 0.013\ rad$$

$$S_A = \frac{h_{obs}}{tan(\gamma)} = \frac{35}{tan(0.013)} = 2692.1\, ft$$

$$S_{T/O} = S_G + S_A = 6171.5 + 2692.1 = 8864\, ft.$$