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Vibrational Analysis of a Wing with Morphing/Variable Winglets.

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AER870 Aerospace Engineering Thesis – Final Report

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Abstract

This work presents a numerical and analytical vibrational and structural analysis of a wing model with morphing winglets. The methodology for this analysis is developed by the undergraduate thesis students in collaboration with the faculty at Toronto Metropolitan University (formerly known as Ryerson University) and performs a static structural and modal analysis for the developed wing with morphing winglet model. This report summarizes the modal and static structural analysis conducted on the model. The CAD model for the study was developed on Solidworks while the two analyses were conducted on Ansys. The wing model was developed with four winglet configurations with varying cant angles. Various winglet configurations were designed with variable cant to study the effect of morphing winglet on the structure's modal analysis. During the structural test, it was observed that the model undergoes a deflection of 0.001 in under the prescribed loading conditions. The modal analysis was completed for the four configurations using similar boundary conditions as for the static structural analysis. The average frequency for the first mode shape for all the configurations were recorded to be 29 Hz.

The results for the modal analysis conducted on Ansys were then verified using an analytical methodology. The equations for this method were developed by modelling the wing as a slender and straight wing with rigid root and no sweep back angles. The procedure for this accounted for the maximum speed of the wind tunnel at Toronto Metropolitan University. The frequency solution obtained from the analytical methodology was then compared to the frequency obtained from the modal analysis completed on Ansys. The frequency at flutter was calculated to be 30 Hz. This validates the assumptions and expectations made prior to both methods of analysis.

The next step for this study would be to build the 3D model of the wing with morphing winglets and conduct a ground vibration testing and wind tunnel testing. The observations made during this study, specifically, the natural frequencies, damping, and mode shapes, can be further compared and validated through conducting the ground vibration testing and wind tunnel testing of these models. By comparing the results, it is possible to validate the numerical models and simulation techniques used for the modal analysis and improve the design of the wing with winglets.

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1. Introduction

The development of novel aircraft designs that are more fuel-efficient, and environmentally friendly has been an active area of research in the aerospace industry. Morphing wing technology, which allows for the adaptation of the wing geometry during flight, has been identified as a promising approach to achieving these goals. The concept of wing morphing involves altering the shape and configuration of the wing during flight to optimize performance in different flight conditions. In this regard, winglets have been found to improve aerodynamic efficiency by reducing drag, increasing lift, and enhancing stability. The integration of morphing winglets with the main wing has shown promising results in enhancing the overall performance of the aircraft.

In this thesis, we present a study on the vibrational analysis of a wing with morphing winglets. The objective of this study is to investigate the effect of morphing winglets on the modal characteristics of the wing structure. The study will focus on the finite element analysis of the wing structure with morphing winglets using commercial software and wind tunnel experimentation. The results of this study are expected to provide insights into the modal behavior of morphing wing structures and the effect of morphing winglets on wing performance.

The findings of this research could have significant implications for the design and development of more efficient and sustainable aircraft. The results of this study are expected to provide a better understanding of the vibrational behavior of morphing wing structures and the effect of morphing winglets on aeroelastic phenomena such as flutter. Overall, this thesis report aims to contribute to the ongoing research in the field of morphing wings and provide a better understanding of the behavior of wing structures under different flight conditions.

2. Context and Literature Review

2.1 Static and Dynamic Aeroelasticity

Aeroelasticity is the study of the interactions between the aerodynamics, elasticity, and inertial forces of an aircraft. This field is divided into two major sub-disciplines: static aeroelasticity and dynamic aeroelasticity.

Static aeroelasticity deals with the equilibrium response of a structure to aerodynamic loads. Specifically, it is concerned with the study of static deformations, such as wing bending, induced by aerodynamic forces. The focus of static aeroelasticity is to ensure that an aircraft's structure is designed to withstand the loads that it will encounter during operation. In contrast, dynamic aeroelasticity is concerned with the time-dependent behavior of a structure under aerodynamic loads. The study of dynamic aeroelasticity encompasses both the forced and unforced vibrations of an aircraft. Unforced vibrations can arise due to the natural frequencies of the structure, whereas forced vibrations occur when the structure is subjected to external forces, such as gusts or control inputs. The primary focus of dynamic aeroelasticity is to ensure the stability of an aircraft during flight and prevent undesirable phenomena, such as flutter.

One of the most critical aspects of aeroelasticity is the prediction and prevention of flutter, a self-excited vibration of an aircraft that can lead to catastrophic failure. Flutter is a dynamic instability that occurs when the forces generated by the airflow over the wings and other parts of the aircraft cause the structure to vibrate. This vibration, in turn, alters the airflow, causing further structural deformation, and leading to a positive feedback loop that can result in catastrophic failure. A key challenge in aeroelasticity is to design an aircraft that is both stable and efficient. This requires a careful balance between the weight and strength of the structure and the aerodynamic performance of the aircraft. For example, adding weight to the structure to increase its strength can decrease the aircraft's efficiency, while reducing the weight can make the structure too weak to withstand the aerodynamic loads. The study of aeroelasticity is critical for the safe and efficient operation of aircraft. Both static and dynamic aeroelasticity play vital roles in ensuring the stability and performance of an aircraft. Understanding and predicting aeroelastic phenomena, such as flutter, is essential for designing more efficient and sustainable aircraft that meet the demands of modern aviation.

In the aeroelastic analysis of a wing with morphing winglets, the goal is to predict the dynamic response of the structure to aerodynamic loads, including flutter and other self-excited vibrations. This requires a combination of structural dynamics equations and aerodynamic equations, which describe the interaction between the structure and the surrounding fluid. One study that applied this approach was conducted by Kim and Lee (2017), who used a combination of FEA, CFD, and aeroelastic equations to analyze the aeroelastic behavior of a wing with morphing winglets. They found that the morphing winglets reduced the wingtip vortex and increased the critical flutter speed of the wing, demonstrating the potential of this technology for improving the aeroelastic stability of aircraft.

2.2 Morphing Winglet Technology

Morphing winglets have emerged as a promising technology for enhancing the aerodynamic performance of aircraft. These winglets are designed to adapt the wing geometry during flight to reduce drag, increase lift, and enhance stability. The integration of morphing winglets in aircraft design has the potential to improve efficiency and reduce emissions. One of the important applications of morphing winglets is in the analysis of aircraft vibration and modal behavior. The vibrational characteristics of an aircraft structure play a critical role in its design, as they determine the natural frequencies and mode shapes of the structure. The mode shapes, in turn, have a significant impact on the dynamic response of the aircraft to aerodynamic loads.

Morphing winglets can affect the vibration and modal behavior of an aircraft in several ways. For example, by changing the wing geometry, morphing winglets can alter the distribution of aerodynamic loads, which can affect the natural frequencies and mode shapes of the structure. This, in turn, can impact the dynamic aeroelastic behavior of the aircraft, including flutter and other self-excited vibrations. Several studies have investigated the impact of morphing winglets on the aeroelastic behavior of aircraft. For example, Kim and Lee (2017) conducted a study on the aeroelastic characteristics of a wing with morphing winglets, using a computational fluid dynamics (CFD) analysis. They found that the morphing winglets reduced the wingtip vortex and increased the critical flutter speed of the wing. Similarly, Hwang et al. (2018) conducted a wind tunnel experiment on a wing with morphing winglets and found that the morphing winglets reduced the wingtip vortex and improved the aeroelastic stability of the wing.

Morphing winglets can also impact the modal behavior of an aircraft, as demonstrated by the study conducted by Menon et al. (2019). They investigated the modal behavior of a wing with morphing winglets using a combination of numerical simulations and experimental testing. Their results showed that the morphing winglets significantly altered the natural frequencies and mode shapes of the wing, which could have implications for the dynamic response of the aircraft. The integration of morphing winglets in aircraft design has the potential to significantly impact the vibrational and aeroelastic behavior of an aircraft. The importance of studying the vibration and modal behavior of an aircraft with morphing winglets cannot be overstated, as it can provide valuable insights into the dynamic response of the aircraft to aerodynamic loads. The results of such studies can be used to develop more efficient and sustainable aircraft designs, contributing to the ongoing research in the field of aeroelasticity.

2.3 Wing Structural and Vibrational Analysis

The structural, vibrational, and aeroelastic analysis of a wing with morphing winglets involves the application of a range of equations and techniques to predict the behavior of the aircraft under various loading conditions.

In the static structural analysis of a wing with morphing winglets, the goal is to determine the stress and deformation of the structure under a given load. The most commonly used equation for this purpose is the Euler-Bernoulli beam equation, which describes the relationship between the bending moment and the curvature of the beam. This equation is often used in conjunction with finite element analysis (FEA) to model the wing structure and predict its behavior. One study that applied this approach was conducted by Wang et al. (2018), who used FEA to analyze the static structural behavior of a wing with morphing winglets. They found that the morphing winglets reduced the bending moment and deformation of the wing, leading to a reduction in stress and improved structural performance.

In the vibrational analysis of a wing with morphing winglets, the goal is to determine the natural frequencies and mode shapes of the structure. This is typically done using the equations of structural dynamics, which describe the relationship between the stiffness and mass of the structure and its natural frequencies and mode shapes. One study that applied this approach was conducted by Zhou et al. (2015), who used a combination of FEA and structural dynamics equations to predict the natural frequencies and mode shapes of a wing with morphing winglets. They found that the morphing winglets altered the natural frequencies and mode shapes of the wing, which could have implications for its dynamic response to aerodynamic loads.

2.4 Wind Tunnel Testing and Ground Vibrational Testing (GVT)

Wind tunnel testing is a widely used experimental technique for studying the aerodynamic performance and aeroelastic behavior of aircraft structures. In the context of a wing with winglets, wind tunnel testing can be used to study the vibrational and modal characteristics of the structure under various loading conditions. The results of these tests can provide valuable insights into the aeroelastic performance of the structure, contributing to ongoing research in the field of aeroelasticity.

Katz et al. (1993) conducted a wind tunnel study of a wing with winglets to investigate the structural and aerodynamic performance of the structure. The tests included measurements of the wingtip displacement and the wing response to aeroelastic loads. The results indicated that the wing with winglets exhibited lower levels of vibration and aeroelastic instability compared to a wing without winglets, highlighting the potential of winglets for improving the aeroelastic performance of aircraft.

In another study, Hoadley and Maughmer (2000) conducted wind tunnel tests of a wing with winglets to study its modal characteristics. The tests involved measuring the natural frequencies and mode shapes of the structure under various loading conditions. The results indicated that the wing with winglets exhibited a higher first bending mode frequency compared to a wing without winglets, suggesting that the winglets had a significant impact on the modal behavior of the structure. In a more recent study, Zhao et al. (2019) conducted wind tunnel tests of a morphing wing with winglets to study its vibrational and modal characteristics. The tests included measurements of the natural frequencies and mode shapes of the structure under different morphing configurations. The results indicated that the wing with morphing winglets exhibited a higher first bending mode frequency and lower vibration levels compared to a wing without winglets, demonstrating the potential of this technology for improving the aeroelastic performance of aircraft.

Ground vibration testing (GVT) is an important tool for assessing the dynamic behavior of aircraft structures, including wings with winglets. GVT can provide valuable information about the natural frequencies, mode shapes, and damping characteristics of a structure, which are essential for assessing the structural integrity and aeroelastic stability of the wing. Recent literature has demonstrated the importance of GVT in assessing the natural frequencies, mode shapes, and damping characteristics of the wing structure, and its impact on the aeroelastic stability of the wing. The use of GVT allows for an accurate and comprehensive assessment of the structural and aeroelastic behavior of the wing, contributing to ongoing research in the field of aeroelasticity and aircraft design.

In a study by Liu et al. (2021), GVT was used to determine the structural characteristics of a wing with variable-camber winglets. The study investigated the effects of different winglet configurations on the natural frequencies and mode shapes of the wing structure. The results showed that the winglet configuration had a significant impact on the structural behavior of the wing and that the use of variable-camber winglets could effectively reduce vibration and improve the aeroelastic stability of the wing. In another study by Chao et al. (2018), GVT was used to evaluate the aeroelastic stability of a wing with curved winglets. The study investigated the effects of the winglet curvature on the flutter behavior of the wing. The results showed that the winglet curvature had a significant impact on the aeroelastic stability of the wing, and that careful design of the winglet curvature was necessary to avoid potential flutter and aeroelastic instability.

2.5 Instrumentations for Wind Tunnel Testing and GVT

Ground vibration testing is a technical procedure to evaluate the dynamic characteristics of the wing with morphing winglets model. The purpose of this test is to determine the modal characteristics of the structure, such as, damping, stiffness, natural frequencies, and mode shapes to study the dynamic response of the wing with morphing winglets model. The test requires the model to be rigidly fixed at the root. The instrumentations required for the GVT, and wind tunnel testing are as follows:

1. Accelerometers: These are sensors that measure acceleration, which can be used to calculate the displacement, velocity, and frequency response of the structure. Accelerometers are typically attached to the wing structure using adhesives or magnetic mounts.
2. Data acquisition system: A data acquisition system is used to collect and store data from the accelerometers. The system includes analog-to-digital converters, amplifiers, and filters to ensure accurate and reliable data collection.
3. Shakers: A shaker is used to excite the wing structure at various frequencies to measure its response. The shaker is typically attached to the wing structure using clamps or bolts and is driven by a power amplifier.
4. Signal generator: A signal generator is used to generate sinusoidal signals of various frequencies and amplitudes that are used to excite the wing structure.

3. Material Selection

Material selection is a critical aspect of designing and building aircraft structures, including wings with morphing winglets. The choice of material can have a significant impact on the structural performance of the wing, as well as its weight and cost. Therefore, a material selection trade study is an essential step in the design process. The material selection trade study typically involves evaluating various materials based on their mechanical properties, manufacturing process, cost, weight, and other factors. Some commonly used materials in aircraft wing construction include aluminum alloys, composite materials, and 3D printed materials.

Aluminum alloys are lightweight and have high strength-to-weight ratios. They are also relatively easy to work with and can be fabricated using various manufacturing techniques. The spars are made of an aluminum alloy for the purposes of this experimental study. This material was chosen because aluminum is a very versatile metal that has many benefits, including being both lightweight and flexible. When it comes to material selection of the spar, aluminum alloys are the most commonly used material due to their high strength-to-weight ratio, ease of manufacturing, and excellent corrosion resistance. However, wood and steel were also considered that were evaluated in a material selection trade study. Wood is a lightweight and readily available material that has been used in RC Plane construction. It has good mechanical properties, such as a high strength-to-weight ratio and excellent impact resistance. Wood is also relatively inexpensive compared to other materials. However, it has some drawbacks, such as being prone to warping and shrinking. Steel is a strong and durable material that has been used in RC Plane construction for many years. It is resistant to fatigue and has excellent impact resistance. Steel is also a readily available material that can be easily manufactured. However, it is heavy compared to aluminum alloys and other materials, which can result in a heavier wing model. When conducting a material selection trade study, it was important to consider the specific requirements of the aircraft wing, such as weight limitations, operating environment, and manufacturing constraints. For example, for the scope of this experimentation strength and flexibility are some primary concerns, and aluminum alloys may be the best option. Overall, the trade study evaluated the pros and cons of each material option. The results of the trade study are tabulated in the table below. It was determined that the spars fabricated with Aluminum were the best fit for the scope of this study.

Table 1: Spar Material Selection.

	Strength	**Elasticity**	**Manufacturing**	**Total**
Aluminum	10/10	10/10	10/10	30/30
Wood	7/10	6/10	8/10	21/30
Steel	9/10	8/10	8/10	25/30

Wing ribs ad winglets are essential components of the model and provide structural support to the wing surface. The material selection for wing ribs is critical for ensuring the structural integrity and performance of the wing. A material selection trade study between ABS, PLA, and carbon fiber reinforced nylon helped in selecting the best material for wing ribs. ABS and PLA are thermoplastics that are widely used in additive manufacturing, and they offer good mechanical properties such as high strength, stiffness, and toughness. They are also lightweight and relatively low-cost compared to other materials. However, ABS and PLA may not be suitable for high-temperature applications and may be susceptible to environmental degradation over time. Carbon fiber-reinforced nylon is a composite material that consists of carbon fibers embedded in a polymer matrix. Carbon fiber-reinforced nylon offers a high strength-to-weight ratio and excellent fatigue resistance, making it a popular material for aerospace applications. It is also highly resistant to corrosion and can withstand high-temperature environments. However, Carbon fiber-reinforced nylon is typically more expensive than ABS and PLA and requires more specialized manufacturing processes.

When conducting a material selection trade study for wing ribs, it was important to consider factors such as weight, stiffness, strength, manufacturing processes, cost, and environmental conditions. For example, since strength is a primary concern, carbon fiber-reinforced nylon was the best option due to its high strength-to-weight ratio. Overall, the trade study evaluated the pros and cons of each material option. The results of the trade study are tabulated in the table below. It was determined that the ribs and winglets fabricated with Carbon fiber-reinforced nylon were the best fit for the scope of this study.

Table 2:Ribs and Winglets Material Selection.

	Strength	**Elasticity**	**Manufacturing**	**Total**
ABS	8/10	7/10	10/10	25/30
PLA	8/10	7/10	10/10	25/30
Carbon Fiber Reinforced Nylon	10/10	10/10	7/10	27/30

The wing skin is a critical component of the wing model, and its material selection is crucial for ensuring the structural integrity and performance of the wing with the morphing winglets model. A material selection trade study between balsa, Monokote and carbon fiber can help in selecting the best material for the wing skin. Balsa is a lightweight wood that has been used in aircraft construction for many years. It is known for its strength, stiffness, and shock-absorbing properties.

It is also relatively inexpensive and easy to work with. However, balsa is susceptible to water damage and can be prone to warping if not properly sealed and maintained. Monokote is a type of polyester film that is commonly used in model aircraft construction. It is known for its lightweight, low cost, and ease of use. It can be applied to balsa or other lightweight materials to provide a smooth and durable surface. However, Monokote is not as strong or stiff as other materials and may not be suitable for larger or more demanding applications. Carbon fiber is a composite material that consists of carbon fibers embedded in a resin matrix. It is known for its high strength, stiffness, and durability. It is also lightweight and has excellent resistance to fatigue and impact. However, carbon fiber is typically more expensive than other materials and requires specialized manufacturing processes.

When conducting a material selection trade study for wing skins, it was important to consider factors such as weight, strength, stiffness, manufacturing processes, cost, and environmental conditions. For example, if strength is a primary concern, carbon fiber was the best option due to its high strength-to-weight ratio. But since stiffness is a primary concern, Monokote was a better choice. Overall, the trade study helped evaluate the pros and cons of each material option. The results of the trade study are tabulated in the table below. It was determined that the wing skin fabricated with Monokote was the best fit for the scope of this study.

Table 3: Skin Material Selection.

	Cost	**Elasticity**	**Manufacturing**	**Total**
Balsa	8/10	7/10	7/10	22/30
Carbon Fiber	8/10	9/10	7/10	24/30
Monokote	10/10	10/10	10/10	30/30

When conducting a material selection trade study for the spars, skin, and ribs, it was essential to consider the specific requirements and constraints of the project, such as strength limitations, operating environment, and cost. The results of the trade study were used to select the material that best meets these requirements and ensures the structural integrity and performance of the wing with morphing winglets.

4. Static Structural Analysis Data Acquisition and Data Processing

The design of wings for an aircraft involves several key principles and considerations, which are based on a combination of scientific theory, engineering analysis, and practical experience. Some of the key theoretical concepts that underpin wing design include aerodynamics, structural mechanics, and aeroelasticity.

4.1 Airfoil Selection and Winglets Configurations

The principles of aerodynamics are fundamental to wing design. The way in which air flows over a wing affects its lift, drag, and other performance characteristics. The shape of the wing (Airfoil), its angle of attack, and other design features can all be optimized to achieve desired performance outcomes. The SC (2)-0518 airfoil was selected as the design for the wing and the winglets for the purposes of this study. The SC (2)-0518 shape is a widely used airfoil for small to medium-sized aircraft and general aviation. It is a low-drag, high-lift airfoil that is designed to provide good overall performance and handling characteristics. The SC (2)-0518 airfoil has a moderately thick profile, with a maximum thickness-to-chord ratio of 18%. It features a slightly curved upper surface and a flat lower surface, which helps to promote laminar flow over the airfoil and reduce drag. The airfoil also features a small amount of camber, which helps to generate lift at low speeds.

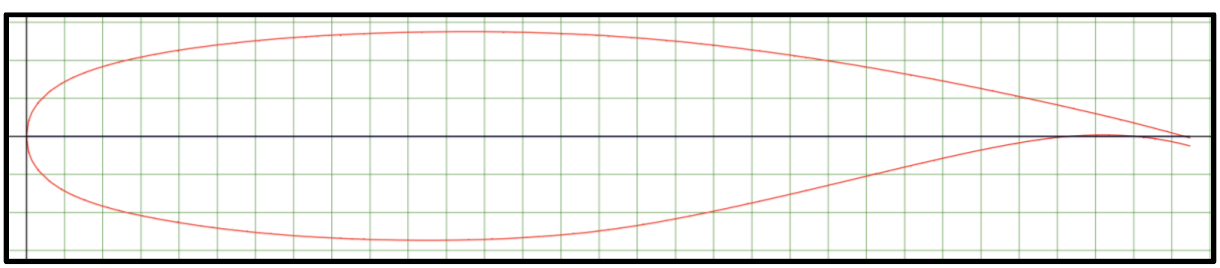

Figure 1: SC (2)-0518 Airfoil Shape.

Table 4: Winglet Configurations.

	Cant Angle	**Toe Angle**
Configuration 1	15	0
Configuration 2	25	0
Configuration 3	35	0
Configuration 4	45	0

4.2 Spar shape Selection

The Spar shape selection was an important consideration in the design of the wing model, as the spar provides the primary structural support for the model. When evaluating spar shape options, a trade study was conducted to determine the best choice for a given application. For this trade study, we compared three spar shape options: a round, square, and flat plate. The weight and strength of each spar shape were evaluated using finite element analysis (FEA). This helped us determine which spar shape provided the best balance of weight and strength for the given application, which is critical for both performance and safety. The manufacturing and maintenance requirements of each spar shape were evaluated to determine which shape is the most cost-effective and practical to produce and maintain over the aircraft's lifetime. Other factors that were considered in the trade study include cost, ease of installation, and any unique performance or operational requirements specific to the given application.

By evaluating these factors for each spar shape option, we made an informed decision on which spar shape is the best choice for the given application. Ultimately, the goal of this trade study was to select a spar shape that provides the best balance of performance, safety, and cost-effectiveness for the given application. In terms of the specific spar shape options being considered, the round shape may offer good overall strength and weight characteristics but may be more difficult to manufacture. The square shape may be easier to manufacture and may offer good stability performance, but it may be heavier and not as strong as the other options. The flat plate shape may be the lightest and easiest to manufacture, and also provide the necessary strength and stiffness for the given application. As a result of the trade study results, the flat plate was chosen as the best design option for the spar shape selection. The results of the trade study are tabulated in the table below. It was determined that the flat plate spar shape was the best fit for the scope of this study.

Table 5: Spar Shape Selection.

	Strength	**Elasticity**	**Manufacturing**	**Total**
Round	10/10	7/10	7/10	24/30
Square	9/10	8/10	8/10	25/30
Flat Plate	8/10	10/10	10/10	28/30

4.3 Structural Analysis

The structural mechanics of a wing must be designed to support the aerodynamic loads and stresses that are placed on the wing during flight. This includes considerations such as materials used in construction, the size and shape of structural elements, and the use of reinforcing elements such as ribs, spars, and stringers. The wing model utilized in this experimental thesis is designed to withstand high stress within a wind tunnel while allowing for moderate deflections to investigate wing oscillations under varying conditions. The wing model is composed of six ribs with the selected airfoil shape and two spars connecting the ribs at 25% chord and 60% chord. The model is then encased in skin made of lightweight, flexible material. The Winglets are constructed from the same airfoil shape in a variety of configurations with varying cant and toe angles. The winglets are made from the same 3D-printed material as the wing ribs. The model's chord and span were determined by constraints such as wind tunnel size. An FEA computational software was used to perform a static structural analysis of the wing model with various winglet configurations. We used the software to select an appropriate material for each component and perform an aerodynamic load and stress analysis.

The CAD model for each configuration was developed on Solidworks. SolidWorks is a popular computer-aided design (CAD) software that can also be used for finite element analysis (FEA) of complex structures such as aircraft wings. However, the static structural analysis of a wing with morphing winglets was conducted using Ansys. This is because Ansys is a more powerful computational tool than Solidworks along with other advantages. Some of these advantages that impacted the computation methodology for this thesis include more advanced materials and large assembly simulation.

To perform structural analysis of a wing with morphing winglets on Ansys, the following procedure was used:

1. The CAD model of the wing with morphing winglets was designed in SolidWorks. Following this it was uploaded as a geometry onto the ansys static structural project.
2. Following this, respective materials for various components were applied using the engineering materials tab. The material properties of the wing and morphing winglets were defined. These properties include modulus of elasticity, Poisson's ratio, and density.
3. A mesh of the model was created using Ansys's meshing tool with a mesh size of 0.25 in. Various mesh sizes were utilized in order to optimize the result convergence for the finite method analysis. The mesh was required to have sufficient density and quality to accurately represent the structural behavior of the wing.
4. Appropriate boundary conditions were applied to the model, including restraints and loads. These boundary conditions were based on the expected operating conditions of the wing.
5. The structural analysis was simulated on Ansys. The software solved the system of equations generated by the FEA and produced results such as stress and deformation in the wing and morphing winglets.
6. The results of the analysis were evaluated to determine whether the design is suitable for the intended purpose.

4.4 Results

The static structural data presented below pertain to the four winglet configurations presented in section 3.1. The static structural analysis results represent the model's maximum deformation and von Mises stress.

4.4.1 Configuration 1 - Cant '15' & Toe '0'.

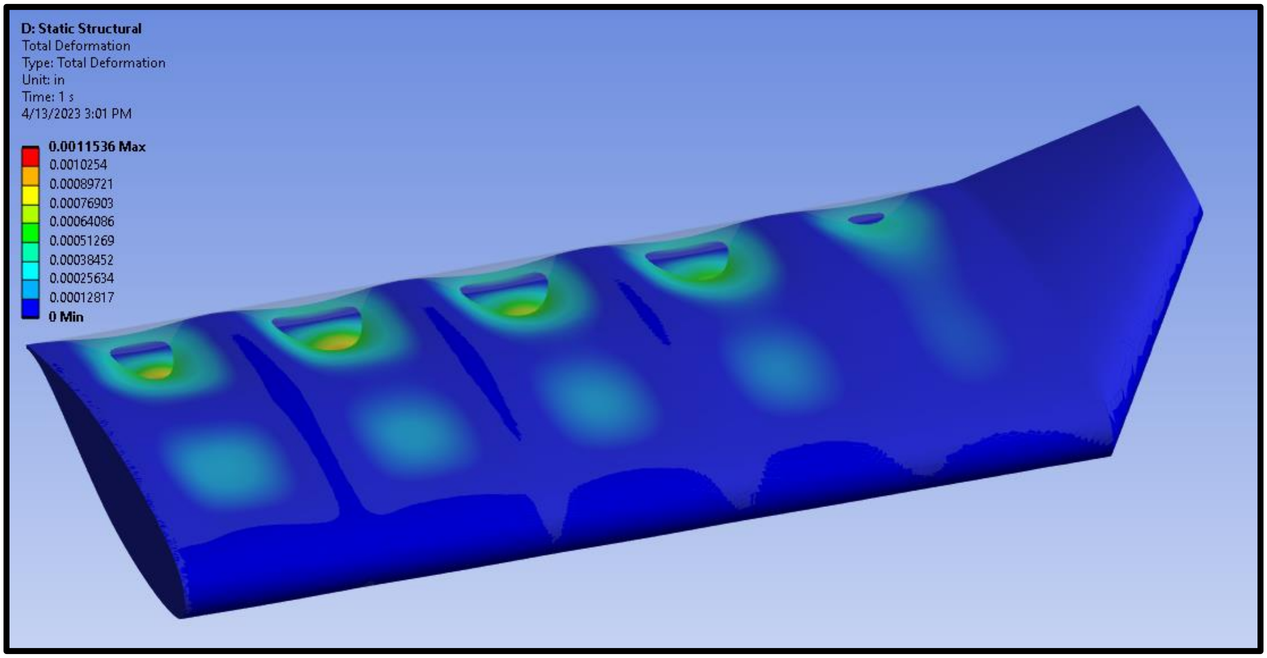

Figure 2: Static Structural - Displacement Results for Configuration 1.

The above figure illustrates the displacement results for configuration 1 with a 15-degree cant angle and 0-degree toe angle. The maximum recorded deformation for the model was 0.001 in.

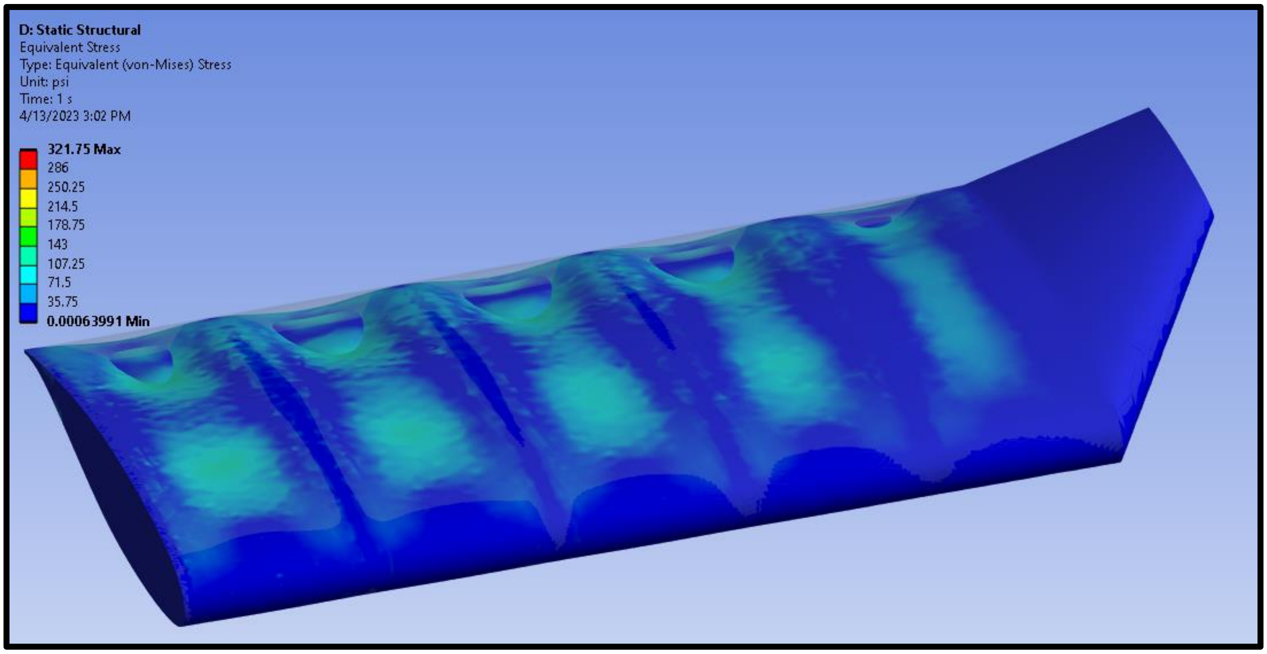

Figure 3:Static Structural - Von Mises Stress Results for Configuration 1.

The above figure illustrates the Von Mises Stress results for configuration 1 with a 15-degree cant angle and 0-degree toe angle. The maximum recorded stress for the model was 322 Psi.

4.4.2 Configuration 2 - Cant ‘25’ & Toe ‘0’.

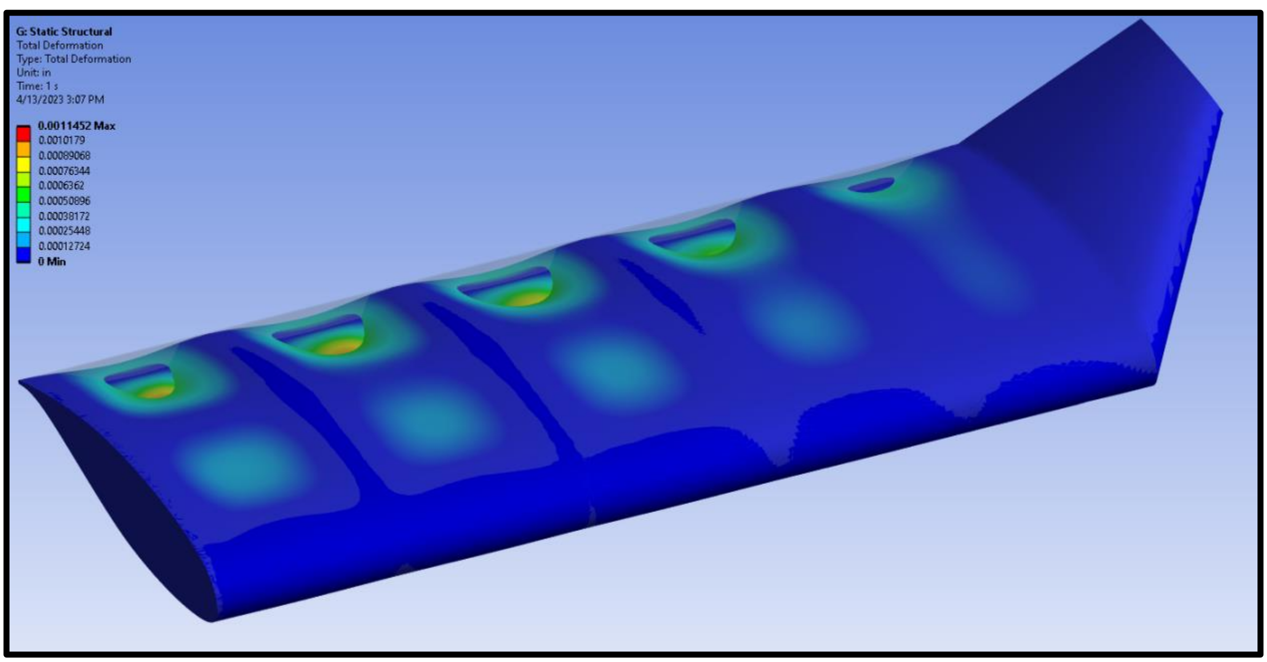

Figure 4: Static Structural - Displacement Results for Configuration 2.

The above figure illustrates the displacement results for configuration 2 with a 25-degree cant angle and 0-degree toe angle. The maximum recorded deformation for the model was 0.001 in.

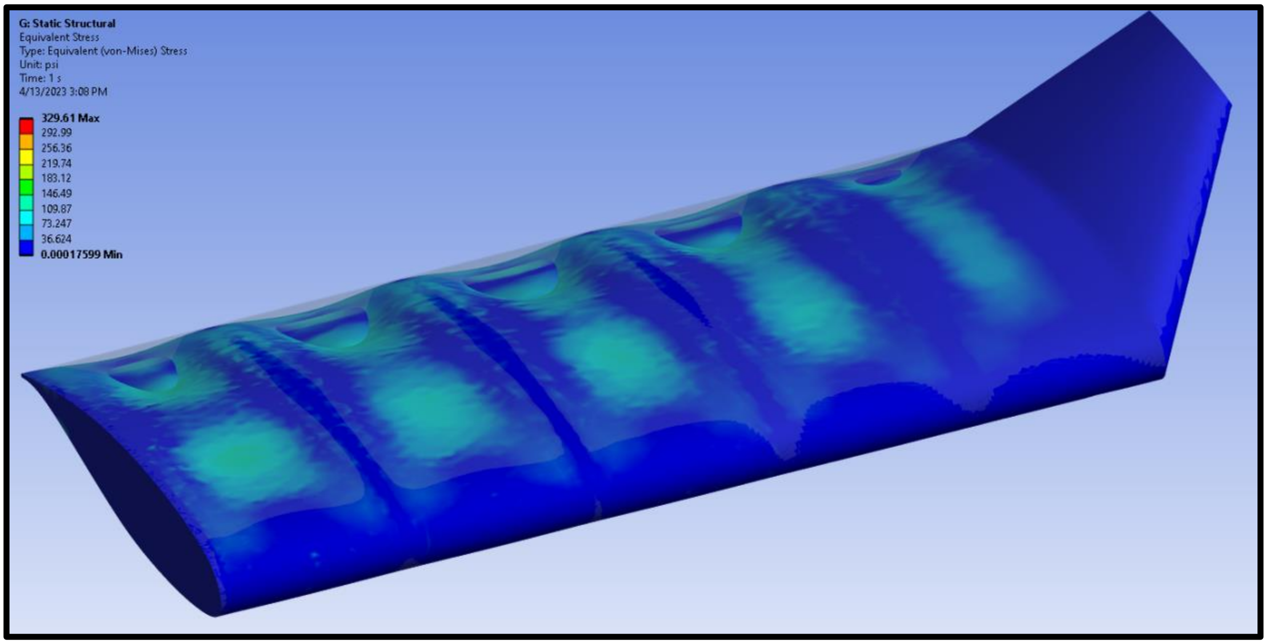

Figure 5:Static Structural - Von Mises Stress Results for Configuration 2.

The above figure illustrates the Von Mises Stress results for configuration 2 with a 25-degree cant angle and 0-degree toe angle. The maximum recorded stress for the model was 330 Psi.

4.4.3 Configuration 3 - Cant ‘35’ & Toe ‘0’.

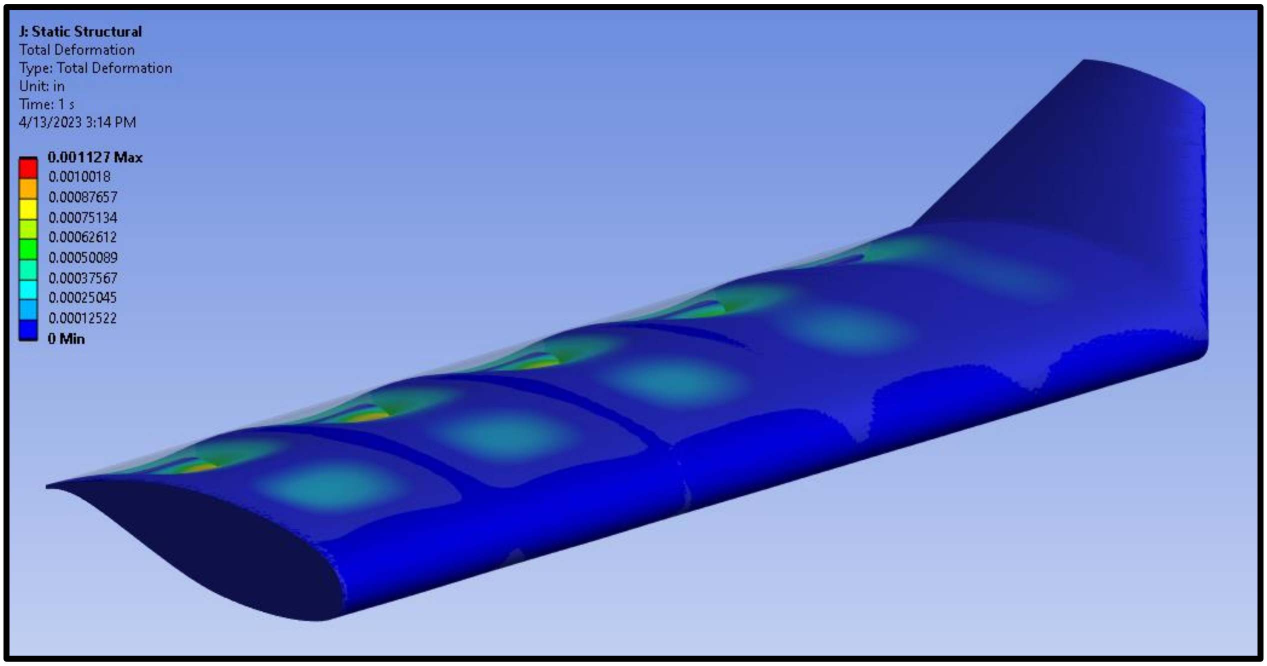

Figure 6: Static Structural - Displacement Results for Configuration 3.

The above figure illustrates the displacement results for configuration 3 with a 35-degree cant angle and 0-degree toe angle. The maximum recorded deformation for the model was 0.001 in.

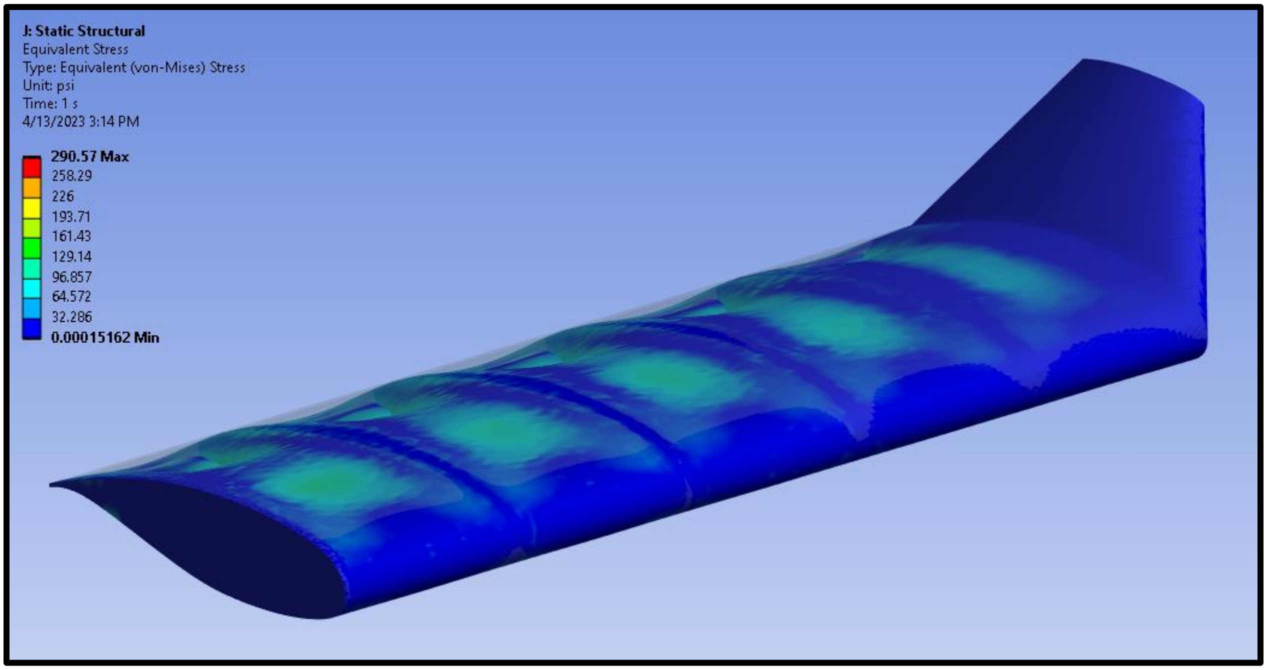

Figure 7:Static Structural - Von Mises Stress Results for Configuration 3.

The above figure illustrates the Von Mises Stress results for configuration 3 with a 35-degree cant angle and 0-degree toe angle. The maximum recorded stress for the model was 290 Psi.

4.4.4 Configuration 4 - Cant ‘45’ & Toe ‘0’.

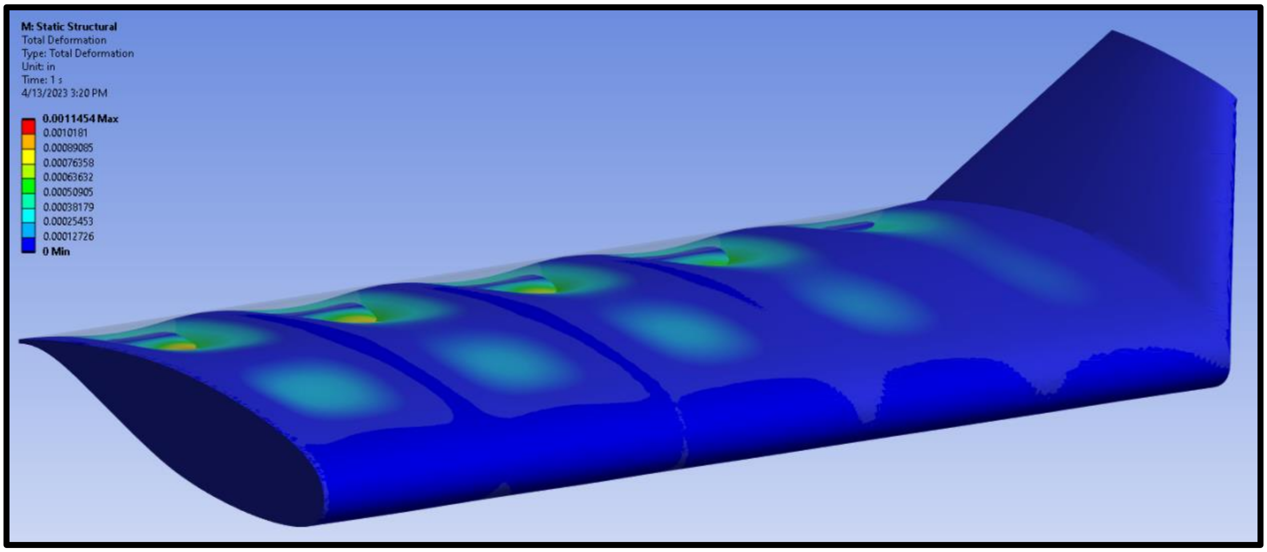

Figure 8: Static Structural - Displacement Results for Configuration 4.

The above figure illustrates the displacement results for configuration 4 with a 45-degree cant angle and 0-degree toe angle. The maximum recorded deformation for the model was 0.001 in.

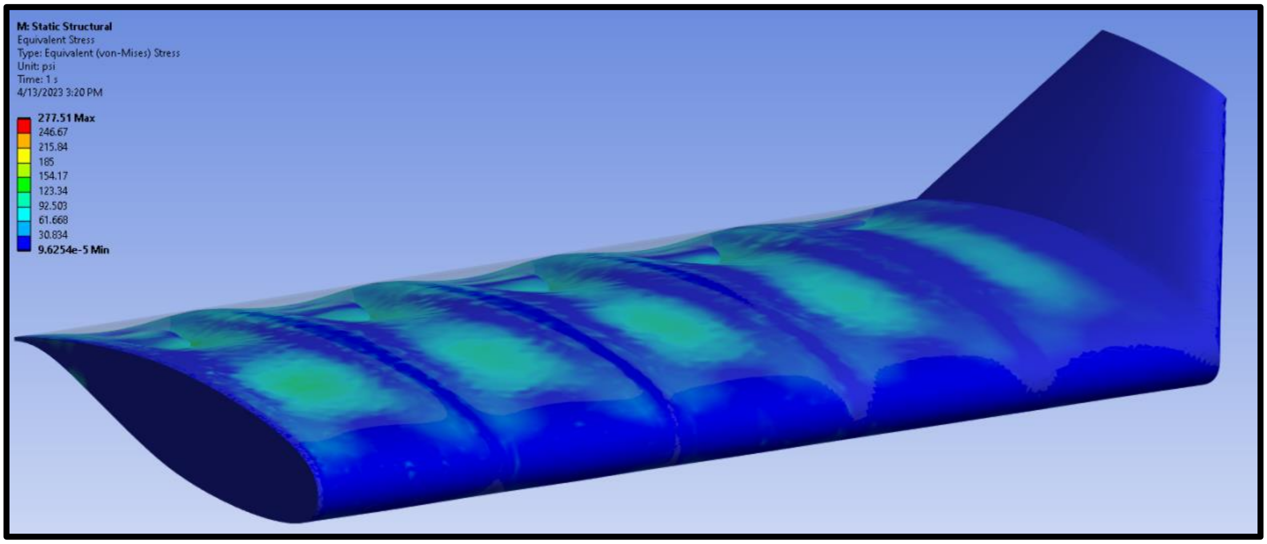

Figure 9:Static Structural - Von Mises Stress Results for Configuration 4.

The above figure illustrates the Von Mises Stress results for configuration 4 with a 45-degree cant angle and 0-degree toe angle. The maximum recorded stress for the model was 277 Psi.

It was observed that the total maximum deformation for the four configurations remained constant while the Von Mises stress followed a downward trend. This result was expected since a similar aerodynamic force was applied for all the wing models. Furthermore, the maximum stress on the model decreased with each configuration. The static structural analysis was completed to ensure that the model can withstand the aerodynamic forces applied to it during the wind tunnel testing. Since the weight of the model isn't a driving design constraint, the model was designed with solid ribs and spars to allow slight deflection while maintaining the model's structural integrity.

5. Modal Analysis Data Acquisition and Data Processing

Modal or vibrational analysis is the study of how a structure such as a wing responds to external disturbances and vibrations. It involves the determination of natural frequencies, mode shapes, and damping ratios of the structure. These parameters are essential in designing and analyzing the structural response of an aircraft to various dynamic loads. The natural frequency of a structure is the frequency at which it vibrates when it is not subjected to any external forces. The mode shape is the pattern of vibration that occurs when the structure vibrates at a particular frequency. The damping ratio is a measure of the rate at which the vibrations of the structure die out over time.

Modal analysis is an essential step in the design and optimization of aerospace structures, including wings with morphing winglets. It allows for determining the structural response of a wing to different types of loads, including aerodynamic, mechanical, and thermal loads. This information can be used to optimize the design of the wing and ensure that it can withstand the expected operating conditions. To perform a modal analysis of a wing with morphing winglets, a mathematical model of the structure was first created. This model was constructed using CAD software and is typically a three-dimensional representation of the wing. Once the mathematical model was created, it was analyzed using finite element analysis (FEA) software. The FEA software allows the natural frequencies, mode shapes, and damping ratios to be determined.

The results of the modal analysis were used to optimize the design of the wing with morphing winglets to improve its structural response to external loads. For example, the results may suggest changes in the thickness or material properties of the wing or modifications to the shape and orientation of the winglets. Ultimately, the goal of the modal analysis was to ensure that the wing is both structurally sound and capable of providing optimal aerodynamic performance.

The Modal/vibrational analysis of a wing with morphing winglets was conducted using Ansys. To perform modal/vibrational analysis of a wing with morphing winglets on Ansys, the following steps were taken:

1. The CAD model of the wing with morphing winglets was imported into Ansys after creating the geometry in Solidworks.
2. A mesh of the model was created using the Ansys meshing tool with a mesh size of 0.25 in. Various mesh sizes were utilized in order to optimize the result convergence for the finite method analysis. The mesh should have sufficient density and quality to accurately represent the structural behavior of the wing.
3. Appropriate boundary conditions were applied to the model, including restraints and loads. These boundary conditions should be based on the expected operating conditions of the wing.
4. The modal analysis was set up on Ansys. This involves selecting the appropriate solver and specifying the number of modes to be analyzed.
5. The model was simulated and analyzed on Ansys. The software solved the system of equations generated by the FEA and produce results such as natural frequencies and mode shapes.
6. The results of the analysis were analyzed to determine whether the design is suitable for the intended purpose.

The modal analysis data presented below pertain to the four winglet configurations presented in section 3.1. The critical mode shapes in modal analysis rely on the particular application and the desired outcomes. The lowest frequency modes, or the first few mode shapes, are often the most significant since they have the most impact on the structure's overall dynamic behavior. The modal analysis results represent the model’s natural frequencies for the first six modes. Moreover, the analysis results also represent the maximum deflection of the model during the case when it is vibrating close to its respective frequency.

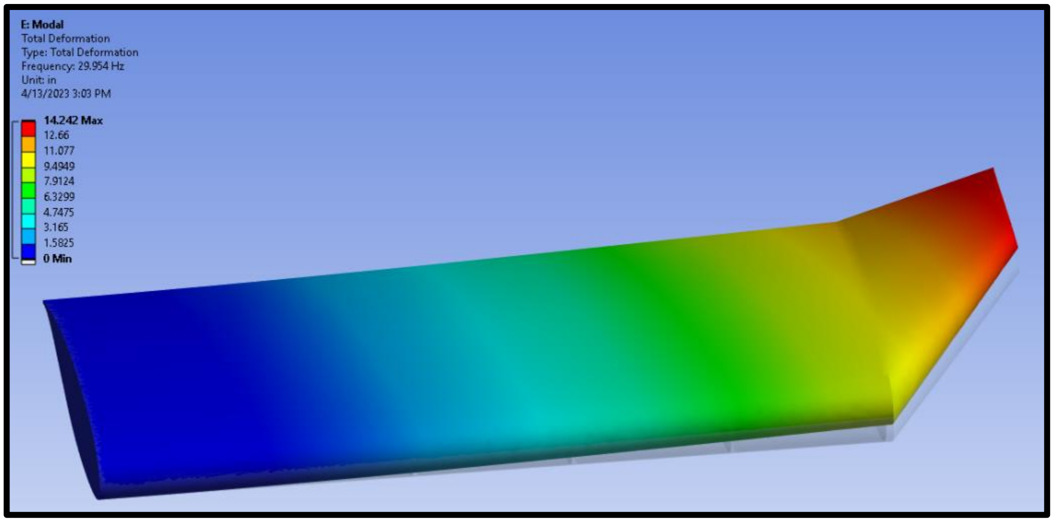

Figure 10: Modal Analysis - for Configuration 1.

Table 6: Mode Shapes and Frequency for Configuration 1.

Mode Shape #	Frequency (Hz)
1	30
2	140
3	184
4	221
5	486
6	532

The above figure illustrates the total deformation completed for the modal analysis of configuration 1. The maximum deformation for the first mode shape was recorded to be 14.2 in right before the model vibrates at its frequency of 30 Hz.

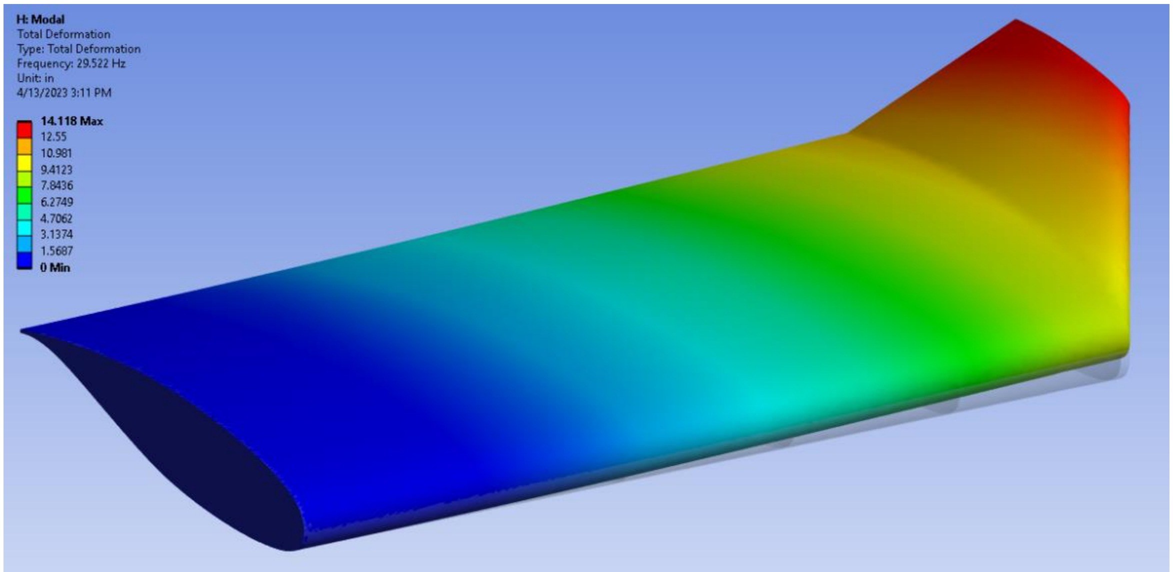

Figure 11: Modal Analysis - for Configuration 2.

Table 7: Mode Shapes and Frequency for Configuration 2.

Mode Shape #	**Natural Frequency (Hz)**
1	29.5
2	136
3	183
4	219
5	465
6	523

The above figure illustrates the total deformation completed for the modal analysis of configuration 2. The maximum deformation for the first mode shape was recorded to be 14.1 in right before the model vibrates at its frequency of 29.5 Hz.

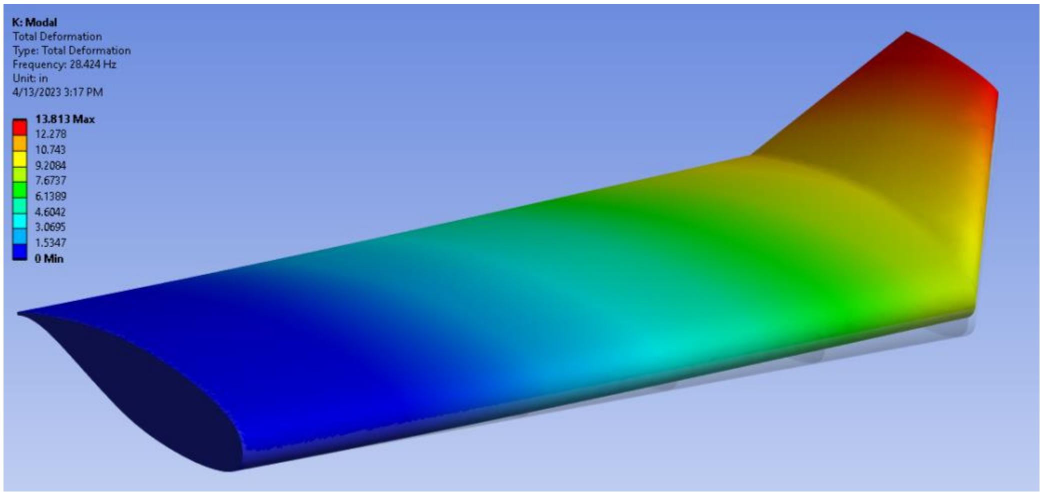

Figure 12: Modal Analysis - for Configuration 3.

Table 8: Mode Shapes and Frequency for Configuration 3.

Mode Shape #	Natural Frequency (Hz)
1	28
2	128
3	180
4	214
5	433
6	524

The above figure illustrates the total deformation completed for the modal analysis of configuration 3. The maximum deformation for the first mode shape was recorded to be 13.8 in right before the model vibrates at its frequency of 29.5 Hz.

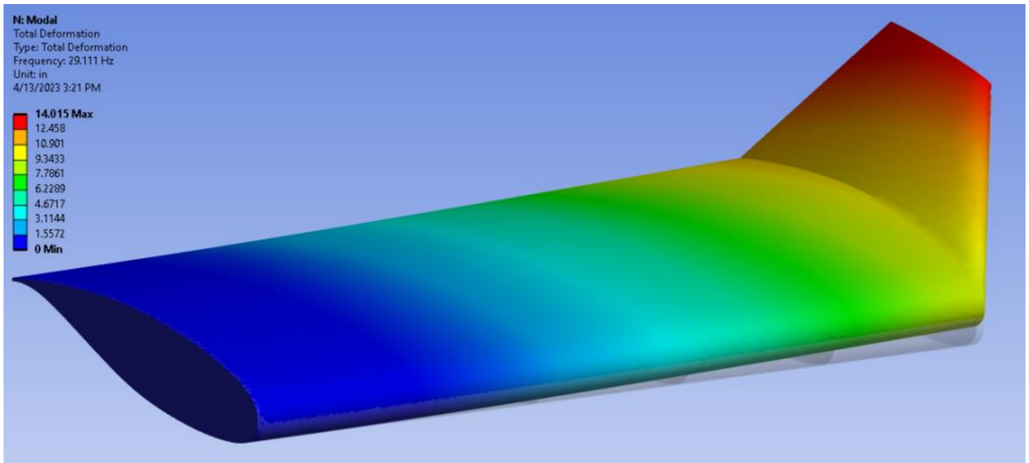

Figure 13: Modal Analysis - for Configuration 4.

Table 9: Mode Shapes and Frequency for Configuration 4.

Mode Shape #	Natural Frequency (Hz)
1	29
2	129
3	184
4	213
5	420
6	529

The above figure illustrates the total deformation completed for the modal analysis of configuration 3. The maximum deformation for the first mode shape was recorded to be 14 in right before the model vibrates at its frequency of 29 Hz.

The natural frequencies of the wing with winglets were extrapolated through modal analysis. The natural frequencies were modified through changing the cant angle of the winglets and the effects of this change on the overall structural response of the wing was investigated. Mode shapes, which are basically the vibrational patterns displayed by the wing with winglets at various natural frequencies, were additionally obtained by modal analysis. The design may be improved by identifying any potential problems with the deformation or structural response of the wing with winglets by analyzing the mode shapes.

6. Analytical Methodology

6.1 Assumptions and Methodology

This section of the report presents the analytical methodology used to validate the results of modal analysis completed using the computational software presented in section 5. The approach uses simplified 2-D flutter analysis for slender and straight wings to extrapolate the frequency and damping for the first mode shape. The assumptions made for this methodology intersects comfortably with the designed wing model for this thesis. The modal characteristics include zero sweep-back angle or natural sweep of the wing, wing fixed rigidly at its root, straight continuous elastic axis which is also perpendicular to the root airfoil. This analytical model is treated as a cantilever beam with freedom to move in bending and torsional directions. The wing model is analyzed separately form the winglet model to achieve higher accuracy of results for modal damping and frequency.

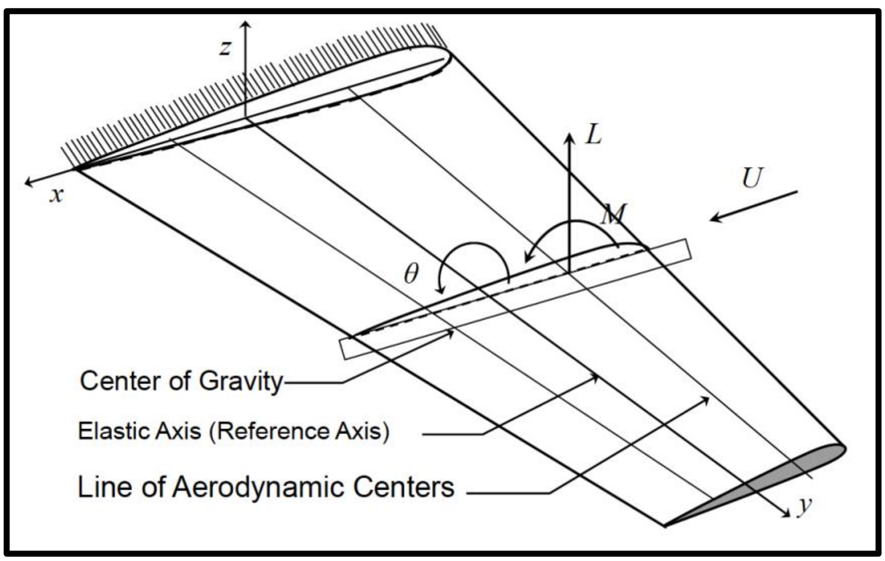

Figure 14: Analytical Wing Model.

To further simplify the problem statement for the flutter analysis of the slender wing, the model can be analyzed as a 2-D rigid airfoil of chord 'c', mass 'm', and moment of inertia 'I'. The airfoil is mounted at a location x1 from the leading edge with a translational spring and a helical spring. For the simplification of the analysis, it is also assumed that the airfoil undergoes zero mechanical damping.

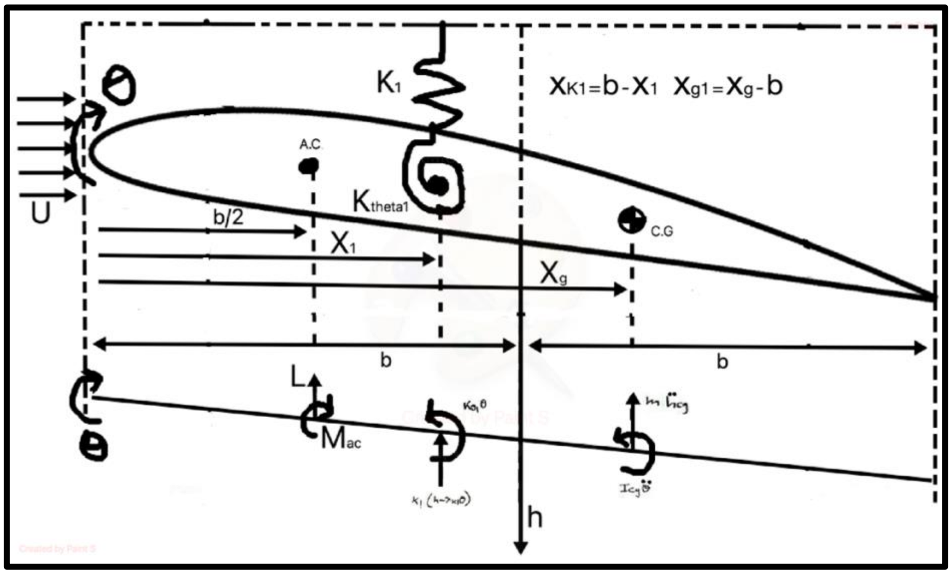

Figure 15: Simplified 2-D Airfoil Analytical Model.

Following this the equations of motion for the system are developed in order to represent the wing model as accurately as possible. The equations represent quasi-steady incompressible aerodynamics, for which Newton's equations, and lagrangian mechanics are used to derive the complete equations of motion to analyze the aeroelastic system. The analysis of the aeroelastic model includes the computation of critical speeds such as flutter speed and studying the effects of the variation of spring coefficients as well as its location along the chord. This variation is important in order to optimize the design for no flutter while obtaining the least natural frequency. This natural frequency is then compared and analyzed with the results of modal analysis completed on Ansys.

Prior to completing the analysis, it was determined that the wing was designed for subsonic wind tunnel experimentations. Therefore, as a design constrain, the maximum velocity in the wind tunnel and for the aeroelastic model were considered to be equal at 25 m/s. Furthermore, it is safe to assume that the incoming velocity is uniform and subsonic. The following table tabulates all the design parameters and assumptions for this methodology.

Table 10: Design Parameters for Analytical Solution.

Parameter	**Value**
k_1	5 N/m
$k_{\theta 1}$	5 Nm/rad
x_1	0.08 m
x_g	0.12
U_{max}	25
Mass	0.08
I_{cg}	$1.4 * 10^{-5}$ kgm^2

Before programming the MATLAB code, it was essential to conceptualize the programming method for easier coding and debugging. The equations of motions in the above figure were utilized to create a function in MATLAB. The input variables for this function were the spring coefficients (k1, and kθ1), location of these springs, and the center of gravity location.

Utilizing the equation of motion and the characteristic equation, the function outputs the critical flutter speed of the model. This function called “critical speeds” and solves the equations for the flutter speed. The figure below illustrates the block diagram of the methodology used to compute and analyze the flutter phenomenon. After calling the function described above, the characteristic equation is used to compute the eigenvalues for speeds 0 to 25 using a for loop. Initially, the program starts with a low velocity that is used to calculate the eigenvalues. The program then checks if all the real parts of the eigenvalues are negative. If the answer is yes, then the speed is increased, and the process is repeated. If the answer is negative, then the flutter has occurred. To make this answer more accurate, the program iterates to the exact solution using smaller increments.

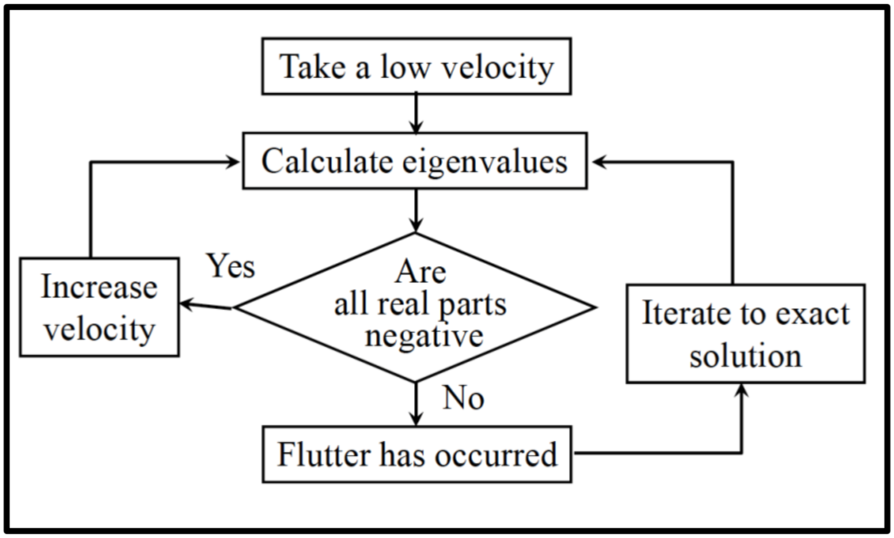

Figure 16: Analysis Methodology.

6.2 Results

The results of this methodology validate the frequency calculated for the first mode shape for the wing models. The calculated flutter speed for the wing model is 58 m/s. The flutter speed calculated is well beyond the range of the wind tunnel currently operational at Toronto metropolitan university. Therefore, it can be cautiously concluded that the designed wing with the various winglet configuration would not undergo flutter during the testing procedure.

The figure below illustrates the variation of damping of the analytical model with respect to speed. The figure follows an expected trend. The system starts with negative damping depicting a stable system. As the velocity increases, the damping of the analytical model increases with it. As the velocity increases and gets closer to the flutter speed, the damping increases rapidly and significantly as it becomes positive. This state of the positive damping for the wing model with winglet represents instability. The damping parameters for this model are calculated by solving the characteristic equation developed for this methodology. The MATLAB code for this can be referred in Appendix A.

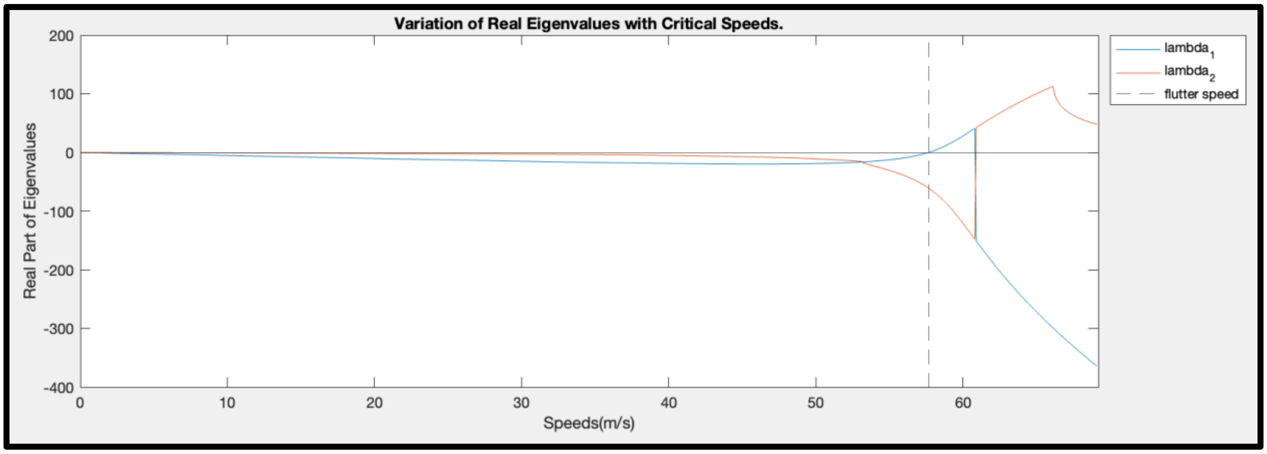

Figure 17: Variation of Damping with Critical Speeds.

The following figure illustrated the frequency of the model in rad/s. As seen in the figure, with the increase in velocity, the two frequency modes converge. As the speed increases and approaches the critical speed (flutter) the frequency of the model decreases. This phenomenon is important as it identifies the model's frequency at flutter. The frequency of the model at flutter was calculated to be at 30 Hz.

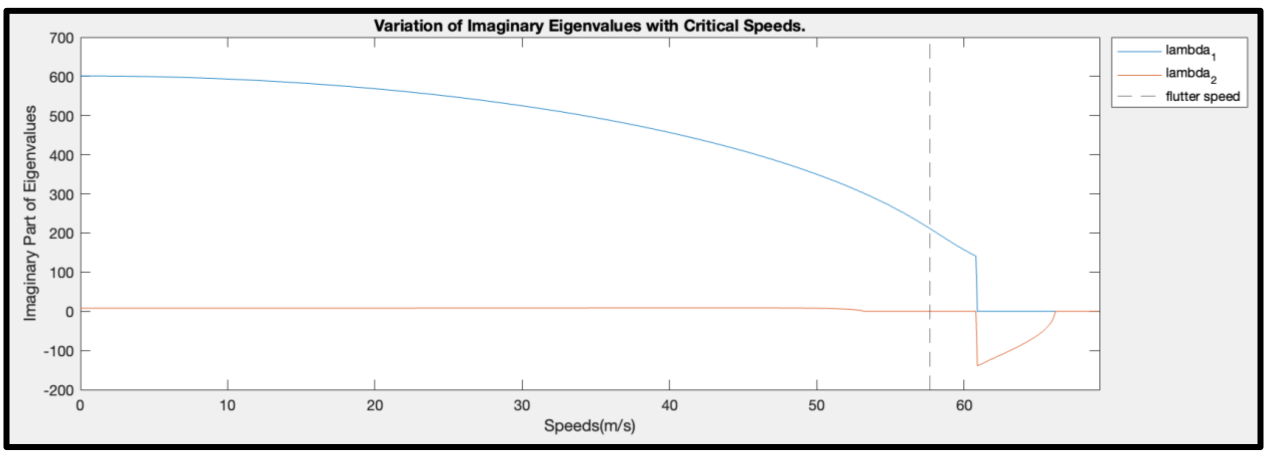

Figure 18: Variation of Frequency with Critical Speeds.

7. Result Validation

The following table compares the frequency computed for the first mode shape for the four configurations in sec. 5 to the frequency calculated from the analytical methodology in sec. 6.

Table 11: Result Summary and Validation.

Test Methodology		**Frequency (Hz)**
Computational Software	Configuration 1	30
	Configuration 2	29.5
	Configuration 3	28
	Configuration 4	29
Analytical Methodology		30

As evident in the above table, the results obtained from the two methodologies validate the assumptions and results of the modal analysis completed on Ansys. Furthermore, these frequencies can be used as a validation tool to compare the results of ground vibration testing and wind tunnel testing. The frequency at flutter extrapolated from the analytical method is 30 Hz. The average frequency right before flutter obtained from the modal analysis in Ansys for the four configurations is 29.125.

The minor difference in the two frequencies account for the assumptions made for boundary conditions. While the boundary conditions must be constructed based on the model assumptions in numerical approaches, they are frequently explicitly provided in the analytical solution. Furthermore, the discrepancies can also be accounted by the accuracy of computational software and the mesh size selected for the process.

8. Future Recommendations

The future tasks for this thesis study are to conduct an experimental analysis in the wind tunnel to analyze the aeroelastic behavior of the wing with morphing winglets model. The procedure for the aeroelastic analysis is comparatively similar to the one presented in the section below. The data acquired from this analysis will be validated to check for any potential issues, such as stress concentrations, deformation, and buckling. Once the ideal data set is verified against the tests performed during ground vibration testing and on Ansys (Static Structural and Modal analysis) the model will be analyzed. The procedure for GVT and wind tunnel testing is presented in the below sections.

By comparing the natural frequencies, mode shapes, and damping ratios acquired from each testing method presented above, these observations could potentially be applied to ground vibration testing and wind tunnel testing. The numerical models and simulation methods used for the modal analysis can potentially be verified by comparing their results.

8.1 GVT Procedure

Ground vibration testing is an essential procedure to evaluate the natural frequencies, mode shapes, and damping characteristics of a wing with morphing winglets model. The following is a procedure for conducting ground vibration testing on the wing model:

1. Set up the test stand: The wing model should be mounted onto a test stand, which is typically composed of a rigid structure.
2. Instrumentation: The wing model should be instrumented with vibration sensors, such as accelerometers, to measure the wing's vibrational responses during the test. The sensors should be placed at strategic locations on the wing surface to capture the mode shapes and natural frequencies accurately.
3. Excitation: An excitation source should be used to induce vibrations into the wing model. This can be done by using an electrodynamic shaker, which applies a force to the wing model at specific frequencies.

4. Test sequence: The test should be conducted in a specific sequence to capture different modes of vibration. The test sequence should start with exciting the lowest frequency mode and then moving to higher frequencies.
5. Data acquisition: During the test, data should be acquired using data acquisition systems that record the signals from the vibration sensors. This data should be carefully synchronized with the excitation signal.
6. Data analysis: After the test is completed, the collected data should be analyzed to identify the natural frequencies, damping ratios, and mode shapes of the wing model. Modal analysis software, such as MATLAB, can be used to process the data and extract the relevant information.

Overall, conducting ground vibration testing on a wing with morphing winglets model is a critical step in evaluating its structural integrity and aerodynamic performance. Proper instrumentation, excitation, and data acquisition, followed by accurate data analysis and interpretation, can provide valuable insights into the model's behavior and help ensure safe and reliable flight.

8.2 Wind Tunnel Testing Procedure

Wind tunnel testing is an essential procedure for evaluating the aerodynamic performance of a wing with morphing winglets model. The following is the procedure for conducting wind tunnel testing on the wing model:

1. Set up the wind tunnel: The wing model should be mounted onto a test section in the wind tunnel.
2. Instrumentation: The wing model should be instrumented with sensors. The sensors should be placed at strategic locations on the wing surface to capture the flow characteristics accurately.
3. Test matrix: A test matrix should be developed that outlines the various test conditions, such as airspeed, angle of attack, and Reynolds number, and winglet configurations to be tested.
4. Test sequence: The test should be conducted in a specific sequence to capture different flow regimes. The test sequence should start with configuration 1 and end with configuration 8.

5. Data acquisition: During the test, data should be acquired using data acquisition systems that record the signals from the sensors. This data should be carefully synchronized with the test conditions.
6. Data analysis: After the test is completed, the collected data should be analyzed to identify the aerodynamic performance of the wing model. Aerodynamic analysis software, such as MATLAB, can be used to process the data and extract the relevant information.

Overall, conducting wind tunnel testing on a wing with morphing winglets model is a critical step in evaluating its performance. Proper instrumentation, test conditions, and data acquisition, followed by accurate data analysis and interpretation, can provide valuable insights into the model's behavior and help ensure safe and reliable flight.

It is considered that the wind tunnel testing would help validate the data acquired for modal analysis using a computational tool and analytical method. Therefore, it is important to determine and then compare the frequencies of the mode shapes to identify the aerodynamic loads operating on the structure and their impact on the dynamic behavior of the structure.

8.4 3D Printing Process

3D printing is a popular and versatile manufacturing process that can be used to create complex shapes and structures with high precision and accuracy. When it comes to creating wing ribs for aircraft, 3D printing can be a great option, especially when combined with carbon fiber-reinforced nylon.

Carbon fiber-reinforced nylon is a composite material that consists of nylon resin reinforced with carbon fibers. It is known for its high strength, stiffness, and durability, making it an excellent choice for creating strong and lightweight wing ribs. The 3D printing process for creating wing ribs with carbon fiber reinforced nylon typically involves the following steps:

1. The first step was to design the wing rib using computer-aided design (CAD) software such as Solidworks.
2. The 3D printer must be set up and calibrated for printing with carbon fiber-reinforced nylon. This may involve adjusting the printer's temperature settings and bed leveling.

3. In 3D printing software, such as Cura, set the optimal number of internal mesh and supports.
4. The carbon fiber-reinforced nylon filament should be loaded into the 3D printer's extruder. The filament is typically fed through a heated nozzle, which melts the material and deposits it onto the build plate layer by layer.
5. The 3D printer creates the wing rib by depositing layers of carbon fiber-reinforced nylon material according to the design specifications.
6. Once the wing rib is complete, it may need to undergo post-processing to remove any support material or rough edges. It may also be necessary to sand or polish the surface to achieve the desired finish.

Overall, 3D printing with carbon fiber-reinforced nylon can be a highly effective method for creating strong and lightweight wing ribs for aircraft. It allows for precise control over the design and manufacturing process, resulting in parts that are tailored to the specific needs of the aircraft. However, it is important to ensure that the 3D printing process is properly calibrated and that the carbon fiber-reinforced nylon material is of high quality to achieve the best results.

8.5 Model Assembly

The assembly process for a wing with morphing winglets for a thesis project will involve several steps, including the assembly of the ribs, spars, and skin. In this case, the ribs will be made of carbon fiber-reinforced nylon, the spars will be made of aluminum, and the skin will be made of Monokote. Following the 3D printing of the ribs and the fabrication of the spars from the chosen materials, the following steps will be taken.

1. Attach the ribs to the spars: Using screws and other adhesive fasteners, attach the 3D-printed carbon fiber reinforced nylon ribs to the aluminum spars.
2. Install the wing tips: Install the morphing winglets onto the ends of the wing by fastening them into the inbuilt lock mechanism.
3. Cover the wing with Monokote: Cover the wing with Monokote, adhering it to the ribs and spars using heat. This process will require cutting the Monokote to size and carefully stretching it to eliminate wrinkles.
4. Test the wing: Conduct a ground vibration test to ensure the wing is safe and functional.

Overall, the assembly process for a wing with morphing winglets involves careful attention to detail and precision in order to create a functional and safe wing. By using carbon fiber-reinforced nylon for the ribs, aluminum for the spars, and Monokote for the skin, this design can provide strength, durability, and flexibility to meet the needs of a variety of flight conditions.

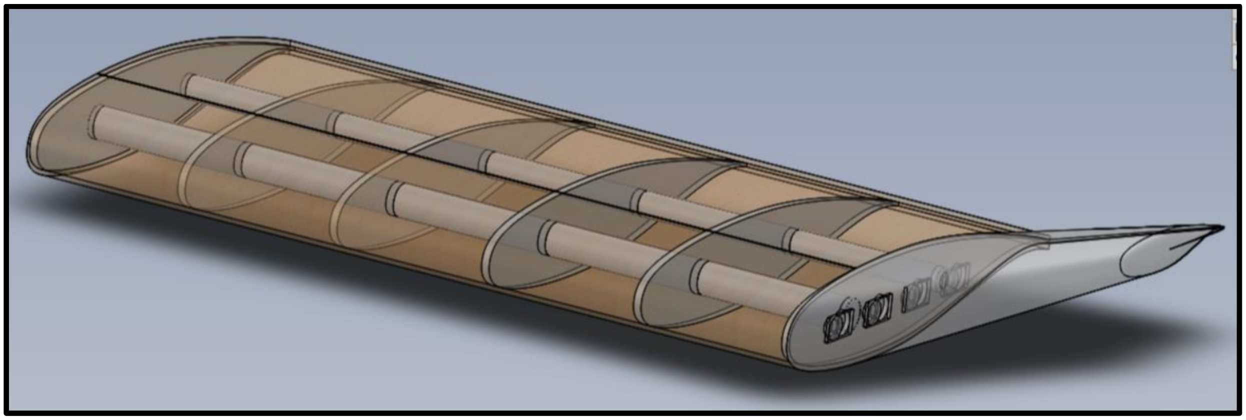

Figure 19: Wing Model Assembly (Configuration 1).

9. Conclusion

Lastly, this dissertation offers an in-depth investigation into the vibrational analysis of a wing with morphing winglets, including a review of the literature on static and dynamic aeroelasticity, wind tunnel and ground vibration testing procedures, and material selection trade studies. The aim for the numerical analysis and analytical analysis is to demonstrate that the morphing winglets are effective at reducing vibration and improving aeroelastic performance. Carbon fiber reinforced nylon for the wing ribs, aluminum for the spars, and Monokote for the skin have also been demonstrated to be a suitable material combination for the wing with morphing winglet model construction.

The results of the static structural analysis show that the wing model is able to withstand the prescribed loading conditions with a deflection of only 0.001 in. The modal analysis also demonstrates that the wing model is able to operate at a frequency of 29 Hz for the first mode shape, which is consistent across all four winglet configurations with varying cant angles. The analytical methodology used to validate the results of the modal analysis is also successful, with the frequency at flutter calculated to be 30 Hz, validating the assumptions and expectations made prior to both methods of analysis.

Moving forward, this study has highlighted the need to build a 3D model of the wing with morphing winglets and conduct a ground vibration testing and wind tunnel testing. By comparing the results of these experimental tests to the numerical and analytical models presented in this thesis, it will be possible to further validate and improve the design of the wing with winglets.
Overall, this study provides valuable insights into the vibrational and structural behavior of a wing with morphing winglets and contributes to the development of advanced aircraft design techniques.

10. References

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10. Appendix A

```
clear
clc
% Given Data (Spring, damping, distances)
k1 = 5;     % In N/m

k_theta1 = 5;  % In Nm/rad
x1 = 0.08;
x_g = 0.12;

syms U lambda

c = 0.31; % in m. chord
b = c/2; % in m. half chord
s = 0.01; % in m. span
rho = 1.225; % density
U_max = 25; % in m/s. max speed
q = 0.5*rho*U^2; % dynamic pressure

% masss and inertia
m = 0.08; % in kg. mass
I_cg = 1.4*10^-5; % in Kg*m^2

% distances between points
%x_k2 = b - x2;
x_k1 = b - x1;
%x_c1 = x_c - b;
x_g1 = b - x_g;

% Matrices
M = [m (m*x_g1); (m*x_g1) (m*(x_g1)^2 + I_cg)];
%B_s = [C_1 (C_1*x_c1); (C_1*x_c1) (C_1*(x_c1)^2)];
B_s = [0 0; 0 0];
B_a = [1 b/2;-b/2 0];
K = [0 1;0 -b/2];
%E = [(k1 + k2) -((k1*x_k1) +(k2*x_k2)); -((k1*x_k1) +(k2*x_k2)) ((k2*(x_k2)^2)
+ (k1*(x_k1)^2) + k_theta1)];
E = [k1 -k1*x_k1; -k1*x_k1 ((k1*(x_k1)^2) + k_theta1)];
f1_U = (2*pi*q*c*s)/U;
f2_U = 2*pi*q*c*s;

rth_eqn = (M*lambda^2 + (B_s + f1_U*B_a)*lambda + (E + f2_U*K));

% Characterestics Eqn.
```

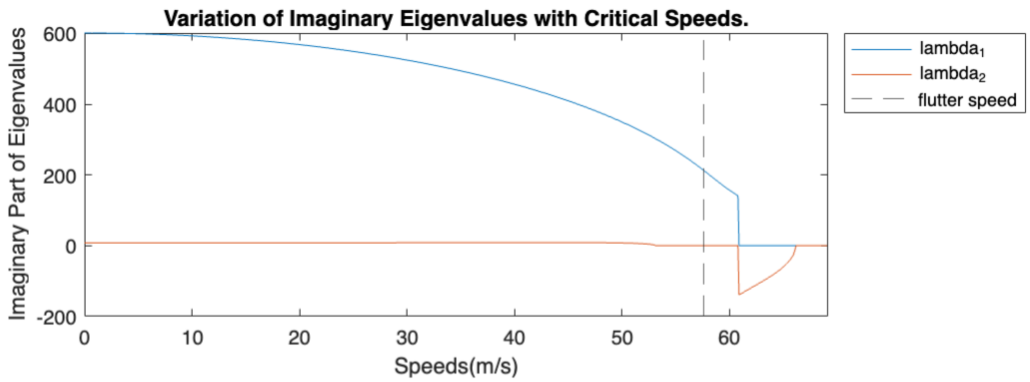
```
det_rth_eqn = det(rth_eqn);
ch_coeffs = coeffs(det_rth_eqn,'lambda');

% Coefficients
p_0 = ch_coeffs(:,1);
p_1 = ch_coeffs(:,2);
p_2 = ch_coeffs(:,3);
p_3 = ch_coeffs(:,4);
p_4 = ch_coeffs(:,5);

% Divergence Speed
U_d = vpa(solve(p_0==0));
U_d = U_d(U_d>0);

% Flutter speed
T_3 = p_1*p_2*p_3 - (p_1^2)*p_4 - p_0*(p_3^2);
U_F = vpa(solve(T_3==0));
U_F = U_F(U_F>0)
```

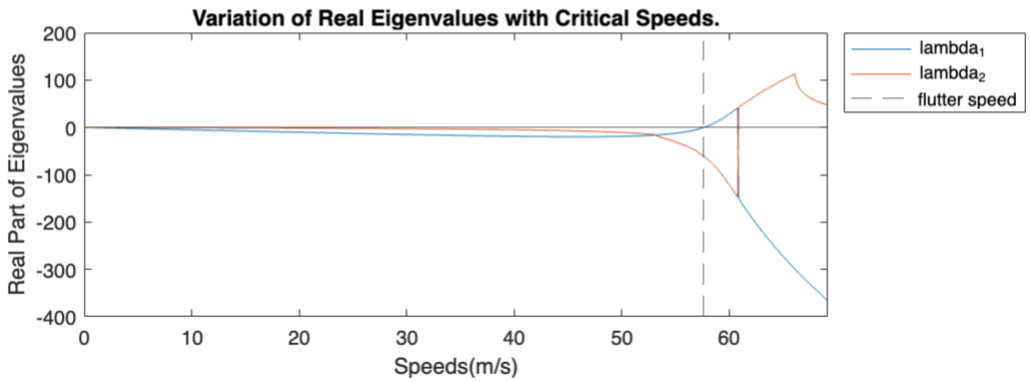
U_F =

57.6558

```
%U_F = U_F(U_F<30);
%%
% Solving the Characterestics eqn
increment = 0.1;
for i = 0:increment:(U_F*1.2)
    r = int16(i/increment + 1);
    char_eqn(r,:) = subs(det_rth_eqn,U,i);
    char_eqn_coeff(r,:) = sym2poly(char_eqn(r,:));
    char_eqn_roots(r,:) = roots(char_eqn_coeff(r,:));
end

%%
% Extracting real and imaginary parts.
roots_real = real(char_eqn_roots);
roots_real1 = roots_real(:,1);
roots_real2 = roots_real(:,3);

roots_imag = imag(char_eqn_roots);
roots_imag1 = roots_imag(:,1);
roots_imag2 = roots_imag(:,3);
frequency_1 = roots_imag1./(2*pi)
frequency_2 = roots_imag2./(2*pi)
```

```
%% Plotting the Variation of Real and Imaginary Eigenvalues with Critical
Speeds.
figure(1)
tiledlayout(2,1)
U_xaxis = 0:0.1:(U_F*1.2);

% top plot - imag
nexttile
plot(U_xaxis,roots_imag1)
hold on
plot(U_xaxis,roots_imag2)
xlim([0 1.2*double(U_F)])
xline(double(U_F),'--')
title('Variation of Imaginary Eigenvalues with Critical Speeds.')
xlabel('Speeds(m/s)')
ylabel('Imaginary Part of Eigenvalues')
legend('lambda_1','lambda_2','flutter speed','location','northeastoutside')

% bottom plot - real
nexttile
plot(U_xaxis, roots_real1)
hold on
plot(U_xaxis, roots_real2)
xlim([0 1.2*double(U_F)])
xline(double(U_F),'--')
yline(0)
title('Variation of Real Eigenvalues with Critical Speeds.')
xlabel('Speeds(m/s)')
ylabel('Real Part of Eigenvalues')
legend('lambda_1','lambda_2','flutter speed','location','northeastoutside')
```


Variation of Imaginary Eigenvalues with Critical Speeds.
Imaginary Part of Eigenvalues
600
400
200
0
-200
0
10
20
30
40
50
60
Speeds(m/s)
lambda1
lambda2
flutter speed


Variation of Real Eigenvalues with Critical Speeds.
Real Part of Eigenvalues
200
100
0
-100
-200
-300
-400
0
10
20
30
40
50
60
Speeds(m/s)
lambda1
lambda2
flutter speed